Correlation Lengths, the Ultrascale, and the Spatial Structure of Interplanetary Turbulence

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Abstract. Characteristic length scales can be employed to describe the large scale structural and statistical features of solar wind plasma turbulence. At least two outer scales of interplanetary turbulence, the correlation scale and an independent quantity that has been called the "Ultra scale," have application in the theory of field line random walk and charged particle diffusion. This paper discusses interpretation of the outer scales, and presents some preliminary observational results concerning their radial evolution.

Introduction

Solar wind plasma and magnetic field fluctuations are broadband, and typically admit a powerlaw inertial range observed in the spacecraft frame at scales corresponding to several seconds up to several hours. If the signal were powerlaw at all frequencies, there would be no inherent scales. However, if a powerlaw has finite extent, there must be scales that characterize this change of behavior. A model for a powerlaw spectrum that flattens at low frequency must have one scale that corresponds to the rollover from flat to powerlaw. A familiar example of this is the exponential correlation function. For solar wind fluctuations, it is apparent that at least two scales are needed to describe the fluctuations, corresponding to the termination of the powerlaw range at both the long wavelength (low frequency) end and at the short wavelength (high frequency) end. Two important questions that have been addressed only partially prior to this time are: How many length scales are required to characterize the fluctuations? What are the physical meanings of these scales? Here we present some preliminary studies of characteristic long wavelength ("outer") scales in the solar wind, motivated in part by considerations of charged particle scattering theory.

Correlation Functions and Correlation Length

The two point correlation function provides a basis for discussion of various characteristic length scales. The correlation function can be defined for the x component of the fluctuating magnetic field $\mathbf{b}(\mathbf{x}) = (b_x, b_y, b_z)$ as

$$R_{xx}(0,0,r) = \langle b_x(x,y,z) b_x(x,y,z+r)\rangle \qquad (1)$$

where it is assumed for convenience that the observation direction is z so that only spatial lags r in the z direction are measured. In the solar wind this would be the radial direction, in accordance with the assumption of frozen-in flow. The most familiar outer scale is the correlation scale, which can defined in accordance with standard practice (2) as

$$\lambda_{cx} = \int_0^\infty dr' R_{xx}(r')/R_{xx}(0), \qquad (2)$$

where we now suppress the irrelevant coordinates. An alternative definition of the correlation scale that is sometime useful when large lag data is unavailable (or unreliable) is the "e-folding" correlation scale which is defined by the condition $R_{xx}(\lambda_{cx}^{e-fold}) = R_{xx}(0)/e$ where $e = 2.71828....$

The integral definition is associated with a familiar interpretation in terms of the power spectrum

$$S_{xx}(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{xx}(r') e^{-ikr'} dr'. \qquad (3)$$

Note that

$$\lambda_{cx} = \pi \frac{S_{xx}(k \to 0)}{R_{xx}(0)} \qquad (4)$$

so that the correlation scale is proportional to the "power at zero frequency," or, more precisely, the power at zero wavenumber in the reduced wavenumber spectrum (7).

The correlation length defined in either of these ways is physically relevant in that it is an estimate of the scale at which the powerlaw gives way to a flat spectrum at low wavenumber. Therefore it provides an estimate for the size of the dynamically important eddies, or "energy containing" eddies in the turbulence. The correlation scale also emerges naturally in the quasilinear theory of field line random walk (8), in that the quasilinear diffusion coefficient (Fokker Planck coefficient) is

$$D_{QL} = \frac{\langle \Delta x^2 \rangle}{\Delta z} = \frac{b^2}{2B_0^2} \lambda_{cx} \tag{5}$$

where B_0 is the mean magnetic field and λ_{cx} is the correlation length of B_x measured in the z direction which is along the mean field. Therefore λ_{cx} it is of importance in particle scattering theory, in which particles can follow field lines as they diffuse.

However important it might be, the correlation scale is an intrinsically very difficult quantity to extract from finite data samples. According to its definition Eq. (2), λ_{cx} is the ratio of the area under the curve of the correlation function to the variance of the magnetic field component of interest, since $R_{xx}(0) = \langle b_x^2 \rangle$. Both of these are explicitly sensitive to unresolved power at low wavenumber. Therefore there is an extreme sensitivity of the correlation scale to contributions from very long period fluctuations in the spacecraft frame. The lowest frequency fluctuations are also crucial in determining (10) whether fluctuations are stationary in time, or homogeneous in space. Discussion of this point invariably focuses on the issue of how one defines the ensemble of interest. It could be appropriate (depending upon the physical problem of interest) that the ensemble be associated with volume averages, or, alternatively, time averages. It might be the ensemble seen by drifting particle distributions. One might also recognize that there a separate ensemble above and below the heliospheric current sheet. These choices raise questions that as yet have not been answered completely, and that most likely have answers that vary from application to application. Even though both the observational and theoretical ambiguities invariably enter into interpretation of the correlation scale, it remains a quantity of considerable, if not central importance, in many problems in space physics.

Is the correlation scale the only important outer scale? If the correlation function were a simple exponential, the answer would be yes, but there is clearly too much complexity at the low frequency end of the observed spectrum to have a simple one parameter dependence (see, e.g., (11)). Thus, the answer seems to be almost definitely not. There are other characteristic large scales in solar wind turbulence, but as yet we do not have a handle on them observationally, and we are only starting to understand the physical nature of such scales in mathematical models. Most likely outer scales are of relevance to both coronal and solar wind dynamics. For example, it is almost certain (11, 16) that very low frequency, long wavelength solar wind observations at 1 AU represent remnant features of solar source surface structures or coronal dynamics below the Alfvén point.

Ultrascale and Zero Crossing Scale

In a nonperturbative approach to the field line random walk (12, 4) problem, the excursion of the field lines in the direction transverse to the mean magnetic field $\mathbf{B}_0$ is not neglected as it is in quasilinear theory. When the fluctuations $\mathbf{b}$ vary in the transverse directions, the correlation properties of the magnetic field in the direction *across* the mean field must enter into the problem. This suggests that field line random walk can involve characteristic scales that are distinct from the usual correlation scale. This expectation is born out by explicit calculation (12) of the field line Fokker Planck coefficient for a model having two components – a slab ingredient (varying along B_0) and a two dimensional ingredient (varying in transverse directions only). For this case the field line diffusion coefficient becomes

$$D = \frac{D_{QL} + \sqrt{D_{QL}^2 + 4D_{2D}^2}}{2}. \tag{6}$$

As in Eq. (5), D_{QL} is the quasilinear result, while D_{2D} is a diffusion coefficient associated with the two dimensional fluctuations,

$$\begin{aligned} D_{2D} &= \sqrt{\frac{1}{2B_0^2} \sum_{\mathbf{K}_\perp} \frac{|\mathbf{b}_{2D}(\mathbf{k}_\perp)|^2}{k_\perp^2}} \\ &= \frac{\tilde{\lambda}}{\sqrt{2}} \frac{\delta b_{2D}}{B_0}, \end{aligned} \tag{7}$$

where δb_{2D}^2 is the variance of the 2D fluctuations, and the Fourier series corresponds to carrying out the calculation in a large periodic box of dimension L in the transverse directions.

The length scale $\tilde{\lambda}$ is defined in terms of an inverse $k_\perp^2$ weighting of the fluctuation spectrum and thus can readily be distinguished from the ordinary correlation scale λ_{cx} (see, e.g., (2)). $\tilde{\lambda}$ depends on the very low wavenumber fluctuations in the transverse direction. For this reason, $\tilde{\lambda}$ has been called the ultrascale (12). It is not difficult to conjure model spectra for which the ultrascale diverges. This can be understood in physical terms. Note that the 2D fluctuations can be expressed as $\mathbf{b}_{2D} = \nabla \times \hat{z} A_z(x, y)$ where A_z is the z-component of the vector potential associated with the 2D fluctuations. Sometimes this is called the poloidal flux function because differences in A_z along a line are equal to the magnetic flux (per unit length) intersecting the line. The mean square vector potential (3) computed over the 2D volume L^2 is

$$\langle A_z^2 \rangle = \sum_{k_\perp} \frac{|\mathbf{b}_{2D}(k_\perp)|^2}{k_\perp^2}. \tag{8}$$

By inspection we see that $\tilde{\lambda} \sim (\langle A_z^2 \rangle / \langle b_{2D}^2 \rangle)^{1/2}$, i.e., $\tilde{\lambda} \sim \Delta A_z / \delta B_{2D}$, where ΔA_z is the r.m.s. magnetic flux in the "magnetic islands" that are characteristic of 2D turbulence (3). Herein lies the key to understanding the ultrascale, as well as physical reasons it might sometimes be divergent. The ultrascale is the length obtained from the ratio of mean square flux to mean fluctuation energy. It is therefore a measure of the mean size of poloidal (2D) flux structures. One should note that the mean size of the poloidal flux structures may be of the order of L, the box size. What happens to the model when $L \to \infty$? The issue of finite $\tilde{\lambda}$ is related to the question of whether all poloidal structures are contained (closed or averaged to zero) within a given box of size L. If so, then for still larger L, the value of $\tilde{\lambda}$ will remain finite. Infinite mean square vector potential $\langle A_z^2 \rangle$ is associated with the infinite box limit whenever there are islands of infinite size that have finite flux.

Another issue is whether there are signatures in the correlation functions that are connected with $\tilde{\lambda}$. Imagine computing a correlation function in the transverse direction using a mean lagged product approach. A probe is placed within a magnetic island, and its signal is multiplied by that of a second probe as the position of the second probe is varied. The averaged signal cannot approach zero until the separation distance is of the order of the island size – otherwise the spatially varying magnetic field will not yet have changed sign relative to the signal at the original position. When the probes are separated beyond the mean island size, contributions to the correlation may reverse sign. On the basis of this heuristic reasoning, we may conjecture that the ultrascale $\tilde{\lambda}$, which is a measure of mean poloidal island size, is also connected with the zero crossing scale of the correlation function. To be precise, let λ_0 be the minimum length for which $R_{xx}(\lambda_0) = 0$. Our conjecture is that $\tilde{\lambda} \sim \lambda_0$. If true this would provide a conceptual and observational basis to discussions of the ultrascale.

Other Outer Scales

In principle there can be many length scales associated with the long wavelength shape of the correlation function. These would be equivalent, for strictly homogeneous turbulence (14), to the scales associated with the behavior of the spectrum as $k \to 0$, e.g., the coefficients in a power series expansion about $k = 0$. Real turbulence that is not so well behaved mathematically at extremely large scales, will also have other characteristic lengths besides the correlation scale and the ultrascale. In the solar wind, these additional outer scales include at least two.

First, there is a scale associated with the breakdown of the assumption of local homogeneity. Fluctuations larger than some scale cannot with reliability be described by homogeneous turbulence theory, nor is it likely that even the descriptive framework of spectra and correlation functions is appropriate for such structures (see, e.g., (10).) The existence of such a scale is implicit in developments such as WKB theory (15, 1, 5), in which smallness of a scale separation parameter is required. The scale separation parameter of interest for a symmetric radial solar wind is of the order of λ/r for local heliocentric distance r and wavelengths less than λ (6). The scale associated with the breakdown of homogeneity must be of the order of r, and will increase with heliocentric distance. However, it is difficult to pinpoint a single scale at which this breakdown will occur.

A second additional outer scale is of dynamical, rather than kinematic origin. As the solar wind plasma and magnetic field are carried outwards, local dynamical effects spread, destroying memory of the initial data at the solar wind source surface, and generating new *in situ* correlations. The development of an active MHD cascade (17) is the preeminent example of this development. At any given heliocentric distance, an important limitation is that all points in the turbulence cannot communicate with each other instantaneously. Instead, the influence of local turbulence at a point moving in the frame of the mean solar wind spreads at a rate determined by characteristic MHD speeds. In principle, one can calculate the size of this region of dynamical influence about each point in the wind, and thus define a length scale that might be called the "MHD causality length".

To date research in solar wind turbulence has not completely taken into account the nature and influence of the these additional outer scales. Efforts along these lines may help to better understand how local turbulence fits into a more complete picture of the heliospheric plasma.

Observations and Conclusions

We have begun an observational analysis of the outer scales of solar wind fluctuations by using both Voyager 2 and Omnitape data to compute correlation scales and zero crossing scales. Correlation scales are computed according to both integral and e-folding definitions. The Omnitape correlation functions are computed from more than 32 years of data. The results for the correlation function of the N component are shown in Figure 1. Hour averaged Voyager 2 magnetic field data is organized into 27 day samples, and the correlation function is computed for the normal (N) component. For the Omnitape analysis, all at 1 AU, the resulting estimates are averaged over the entire span of the dataset. In the case of Voyager data, results are averaged over samples spanning several solar rotations, providing data points for each quantity that are spaced by several AU. The results are given in the Table below.

Data	R(AU)	τ_c (hours)		τ_0 (hours)
		int	*e-fold*	
Omni N	1	1.97-3.6	2.5	22.4
V2 N	1-4	3.15	4.3	26.9
V2 N	4-7	3.2	5.8	25.9
V2 N	7-10	7.6	7.5	72.5

Table of correlation times and zero crossings, in hours in spacecraft frame. Correlation length and zero crossing scale are obtained by multiplying by a mean solar wind speed ($\approx$ 400 km/s). Correlation time given in both integral and e-folding definition. Range for Omni N corresponds to slightly different analysis routines.

These results indicate that the correlation scale generally increases with heliocentric distance, as has been reported earlier (9). The zero crossing scale also increases but is considerably larger than the correlation scale and is clearly a distinct scale. Measurements of the correlation scale such as these are a central constraints in evaluating dynamical theories of solar wind turbulence (see paper 2.38 these proceedings, and (13)).

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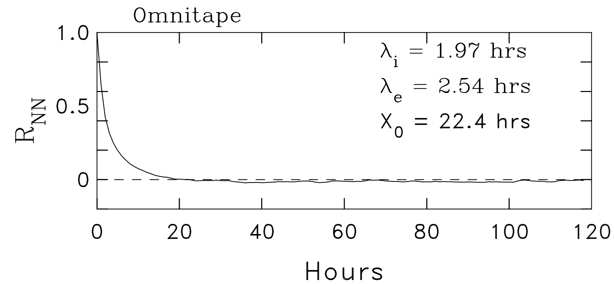

FIGURE 1. Correlation function of the normal component of magnetic fluctuations, from sector-rectified Omnitape data, and associated correlation scale (integral form, λ_i, e-folding λ_e) and zero crossing scale.

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