# Magnetic Reconnection

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## 1. Introduction

About one and a half centuries ago, a solar flare was observed for the first time. It is a truly remarkable and beautiful phenomenon, in fact, one of the most energetic magnetic storms occurring in the solar atmosphere. A major flare may release up to $10^{32}$ erg energy within tens of hours. The source of the flare energy is generally believed to be the magnetic field, and the main way of accomplishing its release is by magnetic reconnection. And reconnection also occurs in the interplanetary space, such as solar wind-magnetosphere interaction, cometary disconnection events, substorms in magnetosphere, etc., and is known as the magnetic field depolarization as well when the aurora and/or substorms are concerned. The magnetic reconnection was also known as magnetic annihilation earlier on and magnetic dipolarization in geophysics.

The magnetotail implies the existence of a current sheet separating two opposite magnetic fields, which is the necessity for the reconnection. In the solar atmosphere, however, the magnetotail-like configuration does not exist in advance since such a configuration is unstable in the corona. But there are three ways in which a current sheet may be formed prior to eruption:

(1) The region near an X-type neutral point can collapse into a current sheet as shown in Figure 1. The magnetic pressure $P$ is balanced by the magnetic tension $T$ in the configuration in Figure 1a. On the other hand, due to the frozen-in effect, the plasma motion may change the magnetic structure as shown in Figure 1b. In $x$ -direction $P$ dominates $T$ , the resultant force $R$ acts inwards, but in $y$ -direction, reverse occurs, the resultant force $R$ acts outwards. This leads to the development of a current sheet in $y$ -direction (Figures 1c and 1d).

(2) When topologically separate parts of a magnetic configuration are pushed together, a current sheet can appear at the boundary between them. This may be the case when either a complex active region field is evolving or new flux is emerging from below the photosphere (Figure 2).

(3) A current sheet may develop when a loss of MHD equilibrium occurs. This process is always associated with the coronal mass ejection, another energetic phenomenon in solar atmosphere (Figure 3).

We will discuss the pure diffusion process of magnetic field in the next section. In section 3, we are going to talk about Sweet-Parker model of magnetic field line reconnection. Petschek's steady-state reconnection model will be discussed in section 4, and we give our conclusions in section 5. Finally, we summarize our work in section 6.

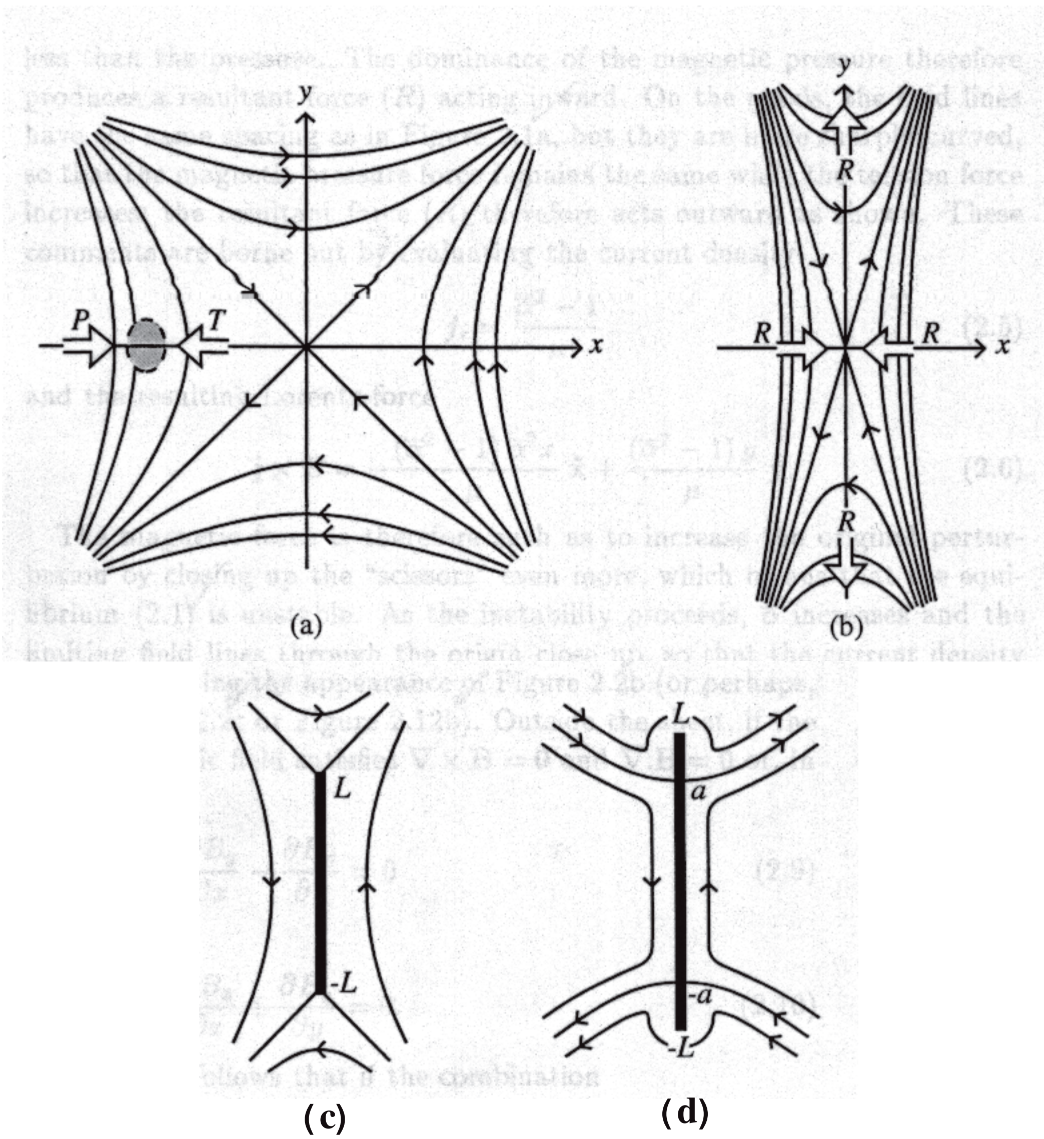


**Figure 1.** Collapse of an X-type neutral point. a). Equilibrium without current sheet. b). The X-type neutral point starts collapsing into a current sheet. c). Y-point configuration with no force acting on the current sheet. d). reverse-current configuration with singularities at the both sides of the current sheet.

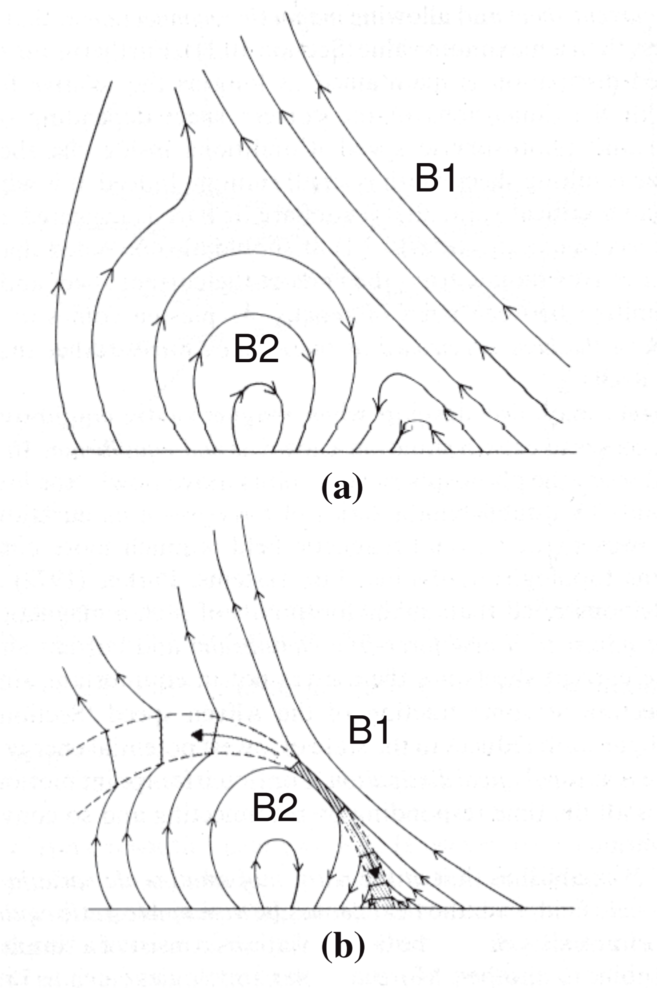


**Figure 2.** An example of *new emerging flux* (NEF). a). The pre-existing (old field), B1, meets a NEF, B2. b). Interacting leads to the formation of a current sheet between them.

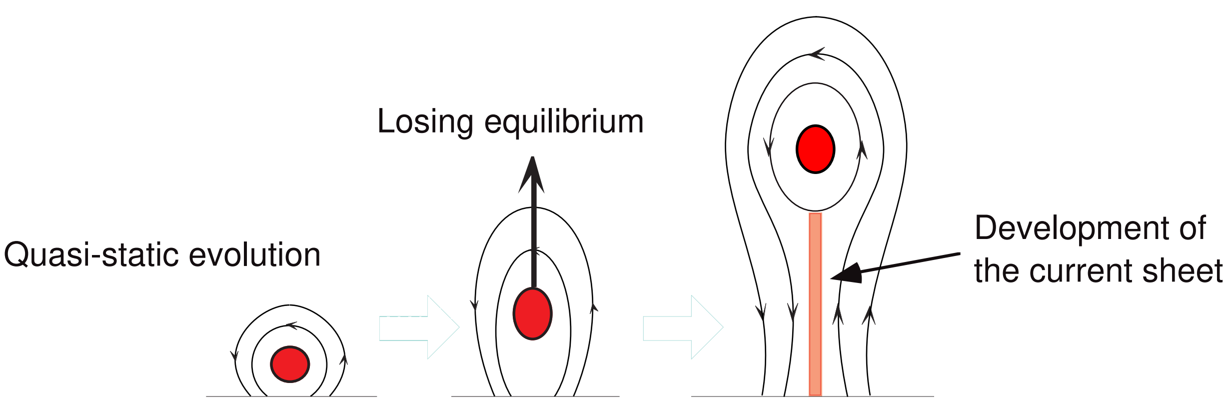


**Figure 3.** The convective motion of the photospheric materials results in the evolution of the coronal magnetic field which roots in the photosphere. As the evolution reaches the critical point, a loss of mechanical equilibrium leads to the development of a current sheet.

# 2. Pure Diffusion Process

To investigate the energy conversion due to the magnetic field line reconnection, let us start with the simplest case of magnetic diffusion as described in Figure 4. The induction equation

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})+\eta\nabla^2\mathbf{B}$$

with magnetic Reynolds number $R_m$ defined as

$$R_m=\frac{L_0 V_0}{\eta}$$

reduces to a simple diffusion equation

$$\frac{\partial \mathbf{B}}{\partial t}=\eta\nabla^2\mathbf{B} \qquad (1)$$

if $R_m$<<1. Where $\eta$ is the magnetic diffusivity which is given by

$$\eta = 5.2\times10^7 \log\Lambda\, T^{-3/2}\ (\mathrm{m^2 s^{-1}}) \qquad (2)$$

for fully-ionized plasma, where $\log\Lambda$ is the so-called Coulomb logarithm with values generally between 5 and 20. Equation (2) describes regular magnetic diffusivity. It is generally believed that $\eta$ should be five orders of magnitude higher in turbulent plasma. But the form of equation remains unchanged. We are not going to solve equation (1) rigorously even though we can with given boundary and initial

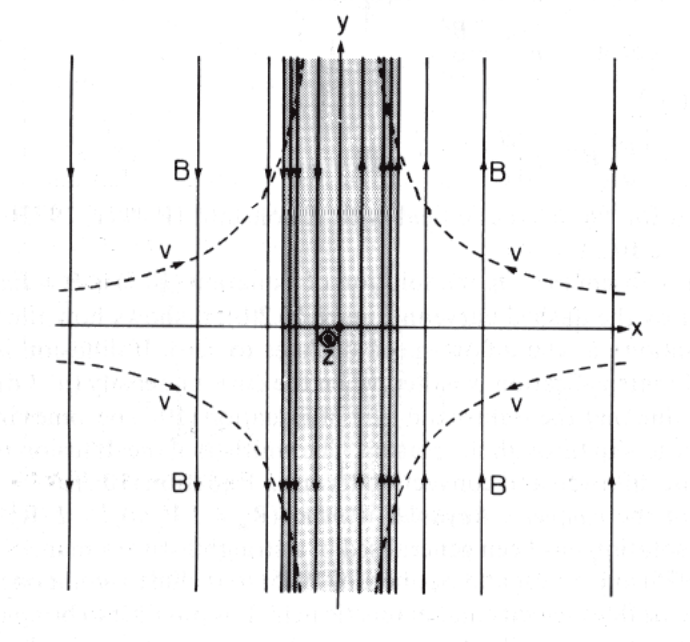


**Figure 4.** Description of magnetic field diffusion. The shaded region supposes to be the diffusion region.

conditions. Instead we are seeking for a scaling law which relates the time scale of energy conversion to the environmental parameters. We have

$$\tau_d = \frac{L_0^2}{\eta} \qquad (3)$$

which implies that a magnetic structure with length scale $L_0$ is destroyed over a diffusion time scale $\tau_d$, and the corresponding magnetic energy is converted into heat by ohmic dissipation. Substituting equation (2) into (3) gives

$$\tau_d = 1.9\times10^{-8} L_0^2 T^{3/2}/\log\Lambda \text{ (s)}. \quad (4)$$

Generally, the typical values of the corona are as follow: $L_0=10^7$m, $T=10^6$; and taking $\log\Lambda=15$, we obtain from equation (4) that $\tau_d=1.28\times10^{14}$s, namely around four million years. This means that the energy conversion due to the simple diffusive process is much too slow to support solar flares. We note that Figure 4 schematically shows that the diffusion occurs in the shaded region, but we have to understand that the diffusion actually takes place in the whole region of interest. Even though anomalous resistivity might occur, the diffusivity $\eta$ in equation (3) is 5 orders of magnitude higher than the normal one, namely the collision diffusivity, and $\tau_d$ given by (3) and/or (4) can thus decrease by 5 orders of magnitude, getting as short as forty years, it is still too long for solar flares. There must exist other energy dissipation processes can account for solar flares.

In practice, the above simple magnetic-field diffusion may be modified in several ways. For instance, the decrease of magnetic field strength near the diffusion region, or known as neutral line, leads to inward magnetic pressure gradients, which drive a flow towards the neutral line from the sides and outwards along it. This requires the inclusion of the convection term in the induction equation and gives rise to a coupling with the equation of motion.

# 3. Sweet-Parker Model

For simplicity, we are only dealing with the steady-state reconnection process, governed by Ohm's Law

$$\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}=\eta\nabla\times\mathbf{B}, \quad (5)$$

the equation of motion

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v}=-\nabla p+\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}, \nabla\cdot\mathbf{B}=0 \quad (6)$$

and the continuity equation

$$\nabla\cdot(\rho\mathbf{v})=0. \quad (7)$$

Most of the solar atmosphere can be considered to be perfectly conducting so that $\eta=0$, equation (5) means that the plasma is frozen to the field lines. This makes a diffusion in a larger region almost impossible. On the other hand, in some local regions, such as a current sheet (Figure 5a), the current density and/or resistivity might be so large that the right hand side of equation (5) is no longer small. Therefore, a breakdown of the ideal-MHD condition can occur in the system, i.e., the magnetic field lines are no longer frozen to the plasma. The field line can thus slip through the plasma, which means that diffusion becomes easier in this region, and the magnetic energy is therefore dissipated into thermal and kinetic energies of plasma. Conceptually we can visualize this process as one during which the magnetic field lines are 'cut', then 'rejoined' to their partners on the opposite side to form a topologically different connection across the current sheet (Figure 5b), and finally separate, together with reconnected plasma, at a relative speed of $2v_A$ (Figure 5c).

Equation (4) shows that the diffusion time scale, $\tau_d$, is proportional to the square of the length scale of the diffusion region, $L_0$. Therefore, if we assume that the energy conversion is confined locally, instead of in the entire active region, we can expect that $\tau_d$ is significantly reduced. In practice the diffusion region possesses a finite length $L$, and there are significant transverse field components (perpendicular to the current sheet). The actual physical process through which reconnection may be initiated is still the subject of active research, and thus no hard and solid conclusions can be drawn yet. Different processes may apply to different applications, and this question is related to the (as yet unsolved) problem of modeling simultaneously and self-consistently both the large and small scale characteristics of the plasmas

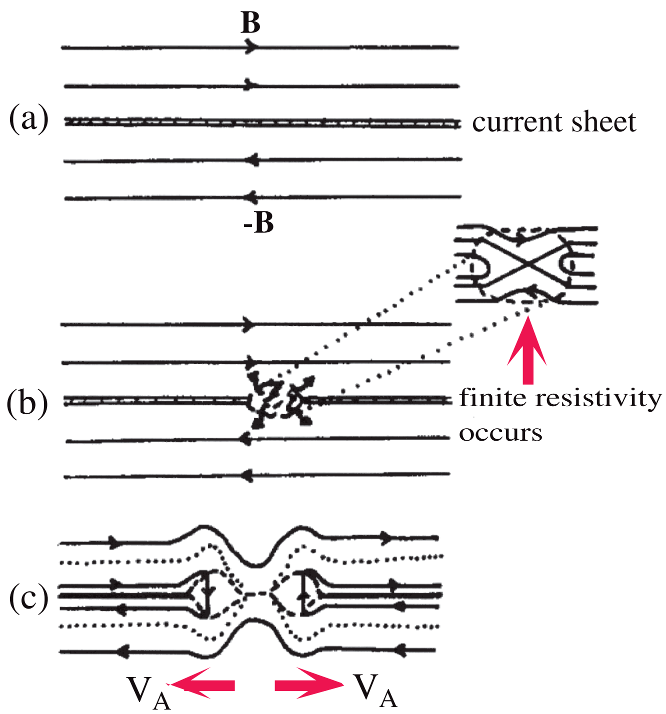


**Figure 5.** Current sheet configuration which is used as a starting point. A current sheet separates anti-parallel magnetic field (a), instability starts developing in a small region in the current sheet, and leads to finite resistivity inside, magnetic field lines are 'cut', then 'rejoined' to their partners on the opposite side (b), and finally separate at a relative speed of $2v_A$ (c).

observed in nature, and the reconnection process in particular.

A simple model for reconnection is the Sweet-Parker model, originally proposed by Sweet (1958) and Parker (1963). The configuration of interest is one in which field lines are carried into the diffusion region (with length of $2L$ and width of $2l$ ) from the sides with speed $v_i$ and out through the top and bottom with speed $v_o$ (Figure 6). Due to the dissipation, the outflow field strength, $B_o$, is less than the inflow strength, $B_i$ ; and for a steady-state, two-dimensional situation, in which the electric field, $E$, is constant, the inflow of magnetic energy (proportional to $EB_o$) exceeds the outflow value (proportional to $EB_i$ ).

As indicated in Figure 6, the inflow region has density, temperature and pressure defined by $\rho_i$, $T_i$, and $p_i$, respectively. The outflow variables are $\rho_o$, $T_o$, and $p_o$. Simple order of magnitude calculations can describe the main characteristics of reconnection in this situation.

In the either region just outside of the diffusion region, we have

$$\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}=0, \tag{8}$$

therefore

$$E_z=\frac{1}{c}v_iB_i\ . \tag{9}$$

According to

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J},$$

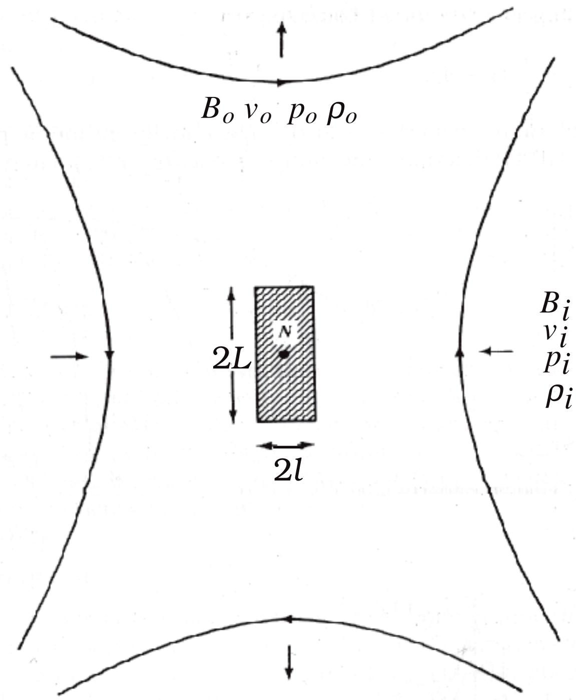


**Figure 6.** A schematic representation of the Sweet-Parker reconnection model. The plasma flows into the diffusion region (shaded), reconnects, and is expelled through the ends. N shows the position of the neutral point.

and considering the fact that $L >> l$, we get

$$\frac{B_i-(-B_i)}{2l}=\frac{4\pi}{c}J_z, \text{ namely}$$

$$\frac{B_i}{l}=\frac{4\pi}{c}J_z\,. \tag{10}$$

Near the current sheet, Ohm's law approximates to $\mathbf{J}=\sigma\,\mathbf{E}$, or $J_z=\sigma\,E_z$, where $\sigma$ is the electric conductivity. By using equations (9) and (10), we thus have

$$v_i=\frac{c^2}{4\pi\sigma l},$$

and the relationship of $\sigma$ to $\eta$ :

$$\eta=\frac{c^2}{4\pi\sigma}$$

leads to

$$v_i=\frac{\eta}{l}, \tag{11}$$

which implies that the inflow velocity must balance the outward diffusion. This defines the thickness of the diffusion region, $2l$, in terms of the inflow velocity.

Mass continuity implies that the amount of plasma flowing into the diffusion region, over the length $L$ in unit time, must equal the mass flowing out through the ends of width $l$. Therefore $\rho_i\,v_i\,L=\rho_o\,v_o\,l$. To simplify matters, consider an incompressible plasma, where $\rho_i=\rho_o$. Thus

$$v_i\,L=v_o\,l\,, \tag{12}$$

giving the outflow velocity in terms of the inflow velocity. Assume that one half of all the magnetic energy that is carried into the diffusion region by the Poynting flux, $c\,\mathbf{E}\times\mathbf{B}/4\pi$, where $\mathbf{E}=-\mathbf{v}\times\mathbf{B}/c$ in the inflow region, is converted into thermal energy, and one half into kinetic energy. Thus, per unit length,

$$\frac{v_i B_i^2}{8\pi} = \frac{1}{2}\,(\rho_i\, v_i) v_o^2, \text{ or}$$

$$v_o = v_A = \frac{B_i^2}{\sqrt{4\pi\,\rho_i}}, \tag{13}$$

which means that the outflow velocity is given in terms of the inflow Alfvén speed. Using equations (11) through (13), the velocity at which the magnetic field is brought into the diffusion region is therefore given by

$$v_i = (\frac{v_o\,\eta}{L})^{1/2} = \frac{v_o}{R_m^{1/2}}, \tag{14}$$

where $R_m$ is the magnetic Reynolds number defined previously. This suggests that the velocity at which the magnetic field, and thus the magnetic energy, is carried into diffusion region is very slow, which implies that the energy conversion cannot be fast. Introducing the Alfvénic Mach number as

$$M_A = \frac{v_i}{v_A} = \frac{1}{\sqrt{R_m}}, \tag{15}$$

which is a dimensionless description of the reconnection rate. Generally, $R_m$ is quite large in astrophysical environment. For example, $R_m \approx 10^{10}$ in the solar corona, and $M_A \approx 10^{-5}$. Therefore the Sweet-Parker model for reconnection is still too slow to account for the energy release in a solar flare. In fact, the energy conversion in this model is mainly realized by means of Joule heating as well although the conversion is now confined in a small region. The time scale of energy conversion is basically characterized by equation (11), namely

$$\tau_c = \frac{l^2}{\eta}, \tag{3.1}$$

which is actually the same as that given by (3) with only one difference that $l \ll L_0$. Generally, $l$ is of order of a thousand kilometers, considering an anomalous resistivity in diffusion region leads to $\tau_c \approx 10^5$s. Although $\tau_c$ reduces a lot comparing with $\tau_d$, it is still long for solar flares. The most serious constraint comes from the dependence of the plasma conductivity $\sigma$ or diffusivity $\eta$. If the anomalous conductivity does not occur, $\tau_c$ even becomes much longer. To increase the efficiency of energy conversion in high-conductivity corona, a mechanism which is almost independent of conductivity is required.

# 4. Petschek Model

Petschek (1964) was the first to propose a fast reconnection model for which the central diffusion region does not need to have a length, $L_e$, of the whole area of consideration, instead the diffusion region occupies a very small length, $L \ll L_e$ (so $l \ll L \ll L_e$), but there are two pairs of shock waves (slow-mode shocks) emanating from the central region, and they may be regarded loosely as current sheet extending from the central current sheet. In a sense, this central current sheet has bifurcated into the two pairs of slow-mode shocks. In the steady-state case, these shocks remain stationary at the positions AOQ and COD, respectively, while plasma carries magnetic flux through them (Figure 7). The direction of the magnetic field vector rotates towards the normal, the strength of the field decreases in the process, and most of the energy is released in the shock waves rather than in the diffusion region. Although the electric conductivity remains high outside of the diffusion region where only a very small part of magnetic energy is dissipated by Joule heating, the main energy conversion occurs at the shocks and depends on the Alfvén speed only. Here, Petschek made two important contributions to reconnection theory. Firstly, he suggested that dissipation is confined to the current sheet (as in Sweet-Parker model), and that the incoming magnetic flux is processed through a small diffusion region, in which the ideal-MHD

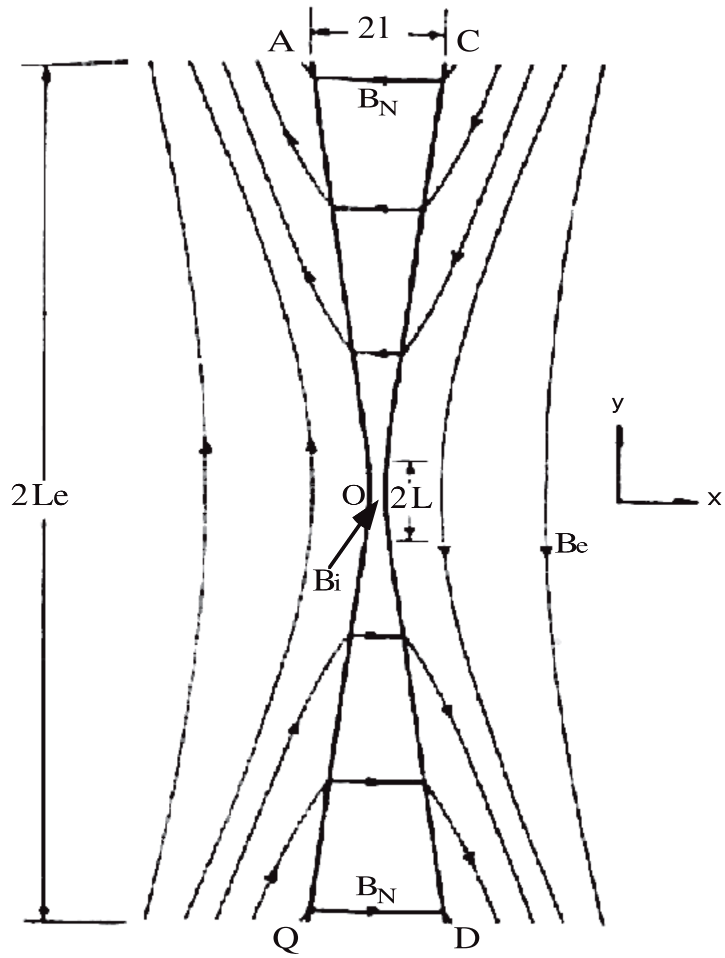


**Figure 7.** Schematic description of Petschek-type reconnection regime. Notice that the density of magnetic field line, and thus the strength of the magnetic field, $B_N$ , in the outflow regions AOC and QOD are much less than the magnetic field, $B_e$ , in the inflow regions AOQ and COD.

approximation breaks down and magnetic field lines are no longer frozen-to the plasma. This breakdown allows a change in magnetic field topology and the establishment of a magnetic field connection across the current sheet. His second contribution was to point out that large-scale or large-amplitude MHD waves should act as the intermediaries that propagate the information about this breakdown into the system at large. Petschek assumed that such waves are generated locally within the diffusion region, although the details of this are still subject to uncertainty. To explain why the shocks provide such an effective mechanism for large-scale energy conversion, we only need to consider that the Alfvén speed, $v_A$, given by equation (13), which is a characteristic propagation speed of slow-mode shocks, is independent of the plasma conductivity $\sigma$.

As shown in Figure 7, Petschek modified the Sweet-Parker model by including the effects of wave propagation. Topologically, Petschek's configuration is consistent with those, such as Sweet-Parker's, containing an X-type neutral point, near which the magnetic field almost vanishes. Here no wave propagates, and diffusion is the main cause of magnetic energy dissipation. With increase of the distance from the neutral point, the magnetic field becomes stronger and stronger, and finally wave propagation dominates the diffusion. Compared with the Sweet-Parker model, the magnetic field lines are significantly curved in the Petschek model. Figure 7 also shows that the density of magnetic field lines in outflow regions AOC and QOD are much lower than that in the inflow region AOQ and COD. This implies that the magnetic field decreases substantially from a uniform value ($B_e$) at large distances to a value ($B_i$) at the entrance to the diffusion region, and then to a much smaller value ($B_N$) in the outflow region, while the flow speed increases from $v_e$ to $v_i$. The principal goals of Petschek's analysis are to investigate the existence of a self-consistent solution, then to determine the maximum value of $v_e$ for a given $B_e$ in terms of the dimensionless Alfvén Mach number $M_e = v_e / v_A$. Petschek treated the diffusion region and inflow region (in which the magnetic field [almost potential] is brought into the diffusion region) separately and matched them at the surface of the diffusion region. To avoid tedious mathematics in this termpaper, we are not going to repeat Petschek's work, instead we directly give some final results. For a more detailed discussion, we refer to Petschek's (1964) original article, and a review by Priest & Forbes (1992).

The effect of the slow-mode shocks is to provide a normal field component $B_N$ which essentially produces the distortion in the inflow field from the uniform value $B_e$ at large distances. When approaching the diffusion region, the field strength falls to

$$B_i = B_e - \left(\frac{4B_N}{\pi}\right)\log\left(\frac{L_e}{L}\right). \tag{16}$$

At the shocks the condition that they be standing is

$$v_e = \frac{B_N}{\sqrt{4\pi\rho}}, \tag{17}$$

and the solution inside diffusion region gives

$$L = \frac{\eta}{2v_A M_e^2}. \tag{18}$$

Finally, by putting $B_i = B_e/2$ in (16) and substituting equations (17) and (18) into (16), Petschek estimates the maximum reconnection rate ($M_e^*$) to be

$$M_e^* = \frac{\pi}{8\log(2M_e^{*\,2} R_m)}, \tag{19}$$

where $R_m$ is the magnetic Reynolds number as defined above. Due to the fact that $R_m$ appears in the logarithm, the effect of electric conductivity on the reconnection rate is much weaker than in the Sweet-Parker model. In practice $M_e^*$ varies typically within the range from $10^{-3}$ to $10^{-1}$, which is much greater than the Sweet-Parker rate. Generally, a solar flare occurs in a region of range between ten and hundred thousands of kilometers, the plasma, together with the frozen-in magnetic field, is carried into in diffusion region at a speed of about $M_e^* v_A$. So, the time scale of the Petschek reconnection process is given by

$$\tau_r = \frac{L_e}{M_e^* v_A}. \tag{3.2}$$

In the coronal environment, $v_A$ is of the order of $10^3$km/s, and $L_e$ is about $10^4$ km for middle class flares and $10^5$ km for very big flares, respectively. The reasonable value of Alfvén Mach number is believed to be 0.1. Plugging these values into equation (3.2) leads to that $\tau_r$ varies from $10^2$ seconds to $10^3$ seconds, which is just needed for the solar flares.

# 5. Conclusions

Petschek's work not only provided a plausible theoretical explanation for solar flares, as well as similar eruptive phenomena in interplanetary space and elsewhere in the universe, but also laid the foundation for substantial theoretical research into magnetic reconnection (e.g. reviews by Vasyliunas 1975; Sonnerup 1979; Pudovkin & Semenov 1985; Forbes & Priest 1987; Heyn *et al.* 1992). Compared with the Sweet-Parker model, two pairs of slow-mode shocks constitute the kernel of Petschek reconnection regime. Although the magnetic energy dissipation in diffusion region is still mainly Joule heating and is still seriously constrained by electric conductivity similar to the Sweet-Parker model, the rate of energy dissipation due to the shocks is only dependent on the local Alfvén speed, the energy conversion can occur at a pretty high speed (as long as it does not exceed the local Alfvén speed). This makes fast energy conversion possible even in high conductivity environment, such as the corona. Furthermore, the inflow of plasma brings typically hundreds of Gauss magnetic field into the diffusion region, inducing an electric field of up to 10 V/cm inside the diffusion region, which is high enough to account for many energetic particles ejected by solar flares. Recent research (Shimada *et al.* 1997) shows that a pair of slow-mode shocks might also be a generator of energetic particles, although it is believed that a single slow-mode shock cannot accelerate particles (Isenberg 1986). In brief, Petschek's model achieves great success in explaining energy conversion of solar flares.

Petschek himself analyzed a steady-state geometry resulting from reconnection between two uniform plasma regions with anti-parallel magnetic orientations. To interpret and explain the reconnection-associated observations in quantitative details, and to also understand theoretically how those two pairs of slow-mode shocks form, more realistic models with more details need to be developed appropriate to each single application.

As part of this objective, Petschek's original analysis has been generalized in two families of steady-state reconnection regimes: (1) 'almost'-uniform reconnection with Petschek's model as a special case, and (2) non-uniform reconnection (e.g. review by Priest & Forbes 1992); and also extended to include the effects of space-time variations in the reconnection rate (Pudovkin & Semenov 1985). More complicated current sheet configuration have been analyzed by Semenov *et al.* (1988); Rijnbeek *et al.* (1991); Semenov *et al.* (1992); Besser (1992); Lin *et al.* (1995), etc. Forbes (1999) even pointed out that not until Semenov and his co-workers developed the 'time-dependent Petschek-type' reconnection model, was the key and essential physics of Petschek reconnection revealed.

# 6. Summary

We discussed some types of energy conversion processes in some astrophysical phenomena, such as solar flares and magnetospheric substorms. We concentrated on the physics of magnetic reconnection processes, which may occur in these phenomena, and investigated the efficiency of energy conversion of different processes. Petschek's reconnection model gives a very plausible energy conversion efficiency. Possible applications and further developments of reconnection theories are also discussed. The most recent work by Lin & Forbes (1999) investigates the role of reconnection in coronal mass ejections (CMEs), and shows that CMEs are impossible without associated reconnection process. The model provides a method for relating the acceleration of the CME to the reconnection rate in a self-consistent manner. More details of magnetic reconnection theories and applications in various fields, such as astrophysics, solar physics, space plasma and laboratory plasma physics, have been reviewed by Priest & Forbes (1999).

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