V.2 Coupling of Energetic Particles to the Plasma (Consequences of transport effects)

Let us now explore one important consequence of the coupling of energetic particle populations to the plasma rest frame. It becomes particularly important for observations of energetic particle populations in moving plasmas, such as the solar wind, as observations are almost always made from a spacecraft at rest. The spacecraft velocity is usually low, except in the case of Solar Probe near perihelion (≈ 300 km/s). Evaluating the effect in observations that we are going to talk about will give us a handle on, whether the particles are coupled and to which rest frame. This is not a trivial exercise, because the particles may be coupled to the cross B drift frame, but not completely to the plasma. The effect is called: Compton-Getting Effect.

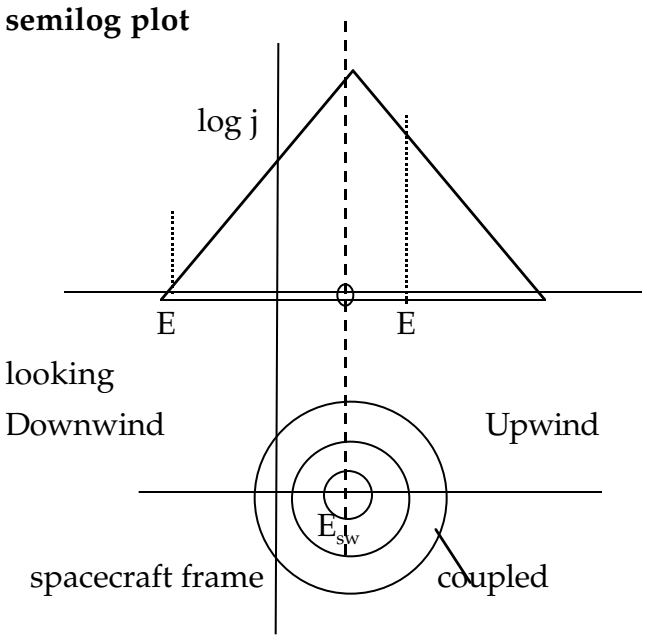

Fig. V2.1: Energetic particle population that is isotropic in the SW frame and an exponential in E. The SW distribution is ignored in this figure.

Let's assume a distribution, which is perfectly coupled to the solar wind. Energetic particle populations often exhibit power law dependencies in differential flux j.

$$j(E) = j_o \left(\frac{E}{E_o} \right)^{-\gamma} \quad \text{V.2_1}$$

or exponentials in energy E

$$j(E) = j_o \, e^{-E/E_o} \quad \text{V.2_1a}$$

These are functions that can be evaluated in analytical form. The first distribution shows a straight line when plotted log-log, the latter when plotted semi-log (Fig. V2.1) . As we can see an instrument on a S/C will see a much higher flux at the same energy E when facing the SW (upwind) than when looking into the opposite direction (downwind). To quantify this behavior we define

the anisotropy

$$A = \frac{j_u - j_d}{j_u + j_d} \quad \text{V.2_2}$$

where u denotes looking upwind and d downwind. (For pickup ions it would be all against nothing: A = 1). An isotropic distribution means A = 0. Let us assume a power law distribution that is isotropic in the solar wind. In velocity space this reads:

$$\boldsymbol{j}(\boldsymbol{v}') = \boldsymbol{j_o} \left(\frac{\boldsymbol{v}'}{\boldsymbol{v}'_{\boldsymbol{o}}} \right)^{-2\gamma} \quad \text{V.2_3}$$

We can perform the transformation into the S/C frame.

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}' + \vec{\boldsymbol{v}}_{\boldsymbol{sw}}$$

Note: we use v′ in the SW and v in the S/C frame. In polar coordinates with the angle Θ between the energetic particle velocity and v_{sw} this reads:

$$v = \sqrt{(v_{sw} + v' \cos\theta')^2 + v'^2 \sin^2\theta'} \qquad \text{V.2_4}$$

$$\theta = \arctan\left[v' \sin\theta' / (v_{sw} + v' \cos\theta')\right]$$

We will evaluate this for θ = 0° and 180°. With some more computational effort the full angular distribution can be transformed. Observing in the same energy band in the upstream and downstream direction in the spacecraft frame, this means we look at particle fluxes in the solar wind frame at the velocities v'_u and v'_d:

$v'_u = v - v_{sw}$ and $v'_d = v + v_{sw}$.

For a flux spectrum that varies as a power law in the SW frame this means we see:

$$j_u = j_o \left(\frac{v - v_{sw}}{v_o}\right)^{-2\gamma} \quad j_d = j_o \left(\frac{v + v_{sw}}{v_o}\right)^{-2\gamma} \qquad \text{V.2_5}$$

with an instrument at energy E (= $mv^2/2$ non-relativistic). The observed anisotropy is:

$$A = \frac{(v - v_{sw})^{-2\gamma} - (v + v_{sw})^{-2\gamma}}{(v - v_{sw})^{-2\gamma} - (v - v_{sw})^{-2\gamma}} \qquad \text{V.2_6}$$

We deal with energetic particles, i.e. mostly we assume v' **>>** v_{sw}. Thus:

$$(v \pm v_{sw})^{-2\gamma} \approx v^{-2\gamma}\left(1 \mp 2\gamma \frac{v_{sw}}{v}\right) \qquad \text{V.2_7}$$

or: $$A \approx \frac{4\gamma v_{sw}}{2v} = 2\gamma \frac{v_{sw}}{v} \left(= 2\gamma \sqrt{\frac{E_{sw}}{E}} \quad \textit{non-relativistic}\right)$$

As can also be seen, the observed distribution obeys only approximately a power law. It rises above the original power lower at low energies for the upstream direction and falls below for the downstream direction, according to the transformation.

How then to reconstruct the distribution in their rest frame from the observation?

We determine γ and A with our instrument on the S/C, assuming that the deviation from the power is small, which is usually true for high enough energies (i.e. E > several 10 keV). We get the appropriate bulk velocity (V_{sw} or V_{sw} • sinα, depending on whether the particles are coupled to the SW or only convected with the magnetic field, see Fig. V2.2) from the other instruments (plasma instrument and magnetometer).

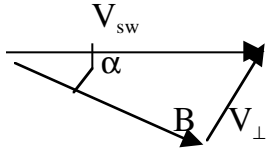

Fig. V2.2: Drift velocity $V_\perp$ with the IMF.

Then we perform the reverse transformation and calculate j_u and j_d for $v'_u = v'_d$. → **[HW #6, Probl. 3b]**

Similar expressions for the Compton-Getting anisotropy may be derived for exponential flux distributions. → **[HW #6, Probl. 3d]**

These transformations may be performed with fluxes for all directions as they are available from instruments, with just some more computational effort. If the distribution is now isotropic in the moving frame, we have shown that this is indeed the rest frame to which the particles are coupled. However, we may uncover remaining anisotropies.

The particles may show

- streaming along B from a source
- anisotropy $\perp$ B due to a gradient

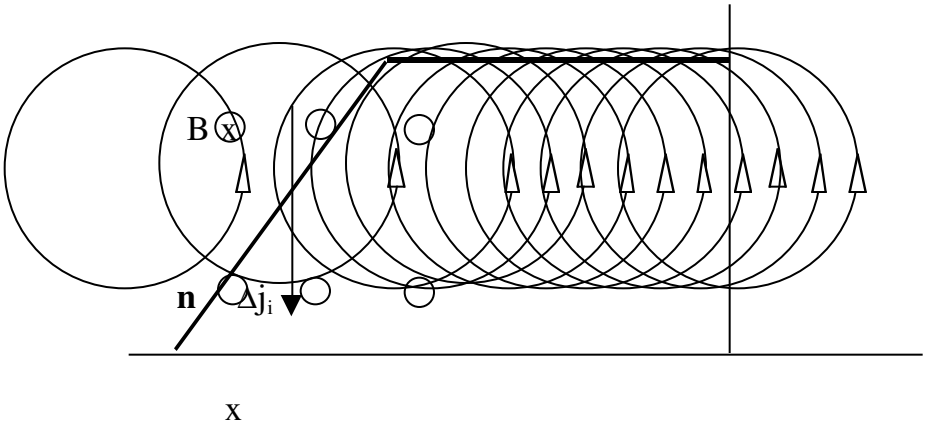

Fig. V2.3: Anisotropy due to a gradient in density n perpendicular to the magnetic field B (pointing into the plane of the figure).

Anisotropies in density gradients are created by the fact that in the region of changing density the balance between the two opposite directions in gyration is not met, as can be seen in Fig. V2.3. The flux difference Δj in the two directions can be described as:

$$\Delta j = 2r_{ci} \bullet \nabla n \bullet v \qquad \text{V.2_8}$$

The typical scale of the gradient $L = n/\nabla n$ must be comparable with or larger than the ion gyroradius to use relation V.2_8. In the case of a sharp density step the flux difference right at the step is:

$$\Delta j = (n_2 - n_1)\, v.$$

This anisotropy is actually observed up to one gyroradius outside the high density region. Thus energetic particles can be used to sense a boundary upon approach.