I.4 Interplanetary Magnetic Field

Up to here we have treated the solar wind and the corona as spherically symmetric. This is a gross negligence.

Viewgraphs (measured Solar Wind Parameters)

As the observation of the corona tells us, it is highly structured, the magnetic field plays an important role. With help of the Zeeman effect detailed maps of the magnetic field in the corona are produced.

Viewgraph (Magnetic fields)

The first picture shows the surface of a very active sun with many sunspots. The magnetic field is highly disturbed and concentrated in the active regions. The other cases represent a more quiet sun with a significant dipole component of the magnetic field. The magnetic field points out of the surface at northern latitudes and into it at southern latitudes. In between the field is more or less across the surface. The regions with the magnetic field lines emerging from the surface are called *Coronal Holes*. These are the regions where the solar wind can emerge easily, as can be seen from the next figure.

Viewgraph (Structure Corona)

The corona presents an image of the magnetic field structure within it. The darker regions are the "Coronal Holes" where the flow of plasma can easily escape the grip of the magnetic field, whereas the bright regions mark trapped coronal plasma in closed magnetic field structures. We call these helmets and streamers. The model on the bottom panel represents a simplified quiet corona. Because the solar wind is a plasma the magnetic field exerts forces on the wind and must structure the wind. Before we start with this new element, let us first spend a little time on the treatment of a plasma in electric and magnetic fields.

Description of Plasma Behavior

The basic equations for electromagnetic fields are Maxwell's Equations:

(in MKSA units)

1) $\nabla \mathbf{E} = \sigma/\varepsilon_o$ Poisson Eqn.

2) $\nabla \times \mathbf{B} = \mu_o \mathbf{J} + 1/c^2\, \partial \mathbf{E}/\partial t$ Ampere's Law

3) $\nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0$ Faraday's Law

4) $\nabla \mathbf{B} = 0$

Maxwell's Equations represent a consistent description of the electric and magnetic fields. This is a closed system in vacuum! However, this is different in the presence of charges. The charge density σ in the Poisson equation produces an electric field **E**
The electric current density **J** in Ampere's Law produces a magnetic field **B**.
These quantities have to be calculated.

This is not trivial in a plasma

E acts on charges
B acts on currents

through the **Lorentz Force**:

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{V} \times \mathbf{B})$$

Therefore, we have to include the behavior of the plasma into the equations. This can be performed with several different levels of complication, depending on the specific problem.

Plasma Descriptions

• trajectories of individual particles

This is a test particle picture. It does not include any modification of background fields by the presence of the plasma. This is not a self-consistent approach, and therefore it is good only as long as the external forces are strong compared with the "home-made" fields of the plasma itself. In essence, the plasma only reacts passive to fields.

In the framework of our course it will be appropriate for the addition of pickup ions to the solar wind plasma and acceleration of a few particles out of a background plasma to high energies.

However, this approach cannot be used for the core plasma itself.
In principle we will have to try to describe the motion **and** the influence of all particles, but this will be an impossible task.

• statistical approach

Used is only the probability for each particle to be at
r: location in configuration space
v: location in velocity space
In a way it is a description for all particles, but with limited information. It explicitly includes dependencies on all other particles.

We use the probability of an arbitrary particle to be at a certain location in phase space

$f(\mathbf{r},\mathbf{v},t)$

which in principle depends on all the other particles. To solve the equation system for this complete distribution function f again amounts to an impossible task so that approximations are made for practical purposes. The distribution function f is used to compute a charge density distribution (which contains implicitly the information of all particles). Interaction between individual particles usually is weak compared to their response to the resulting electric field. At best collisions of particle pairs are considered. This description is used when features of the velocity distribution function f(v) are important for the behavior, as is needed for certain plasma waves.

• fluid approach

In many cases the plasma can just be treated as a macroscopic fluid. I.e. only macroscopic parameters, such as ρ, $\mathbf{v}$, P; $\mathbf{j}$, σ
are used. Such a description is good for large scale phenomena, and therefore it is appropriate for the solar wind. This approach is called the Magneto-HydroDynamic approach (MHD).

We will add a brief discussion of the different approaches and lead with an abbreviated derivation from a general description of the distribution function to the MHD equations.

Boltzmann Equation

Evolution of the (single particle) distribution function f_1 in phase space:

$$\partial f_1/\partial t + d\mathbf{r}/dt \cdot \partial f_1/\partial \mathbf{r} + d\mathbf{v}/dt \cdot \partial f_1/\partial \mathbf{v} = \partial f_1/\partial t$$

$$\partial f_1/\partial t + \quad \mathbf{v} \quad \partial f_1/\partial \mathbf{r} + \quad \mathbf{F}/m \quad \partial f_1/\partial \mathbf{v} = \partial f_1/\partial t$$

$\partial f_1/\partial t$: temporal evolution of f_1 caused by other particles!

$\mathbf{v}\, \partial f_1/\partial \mathbf{r}$: convective variation in configuration space

$\mathbf{F}/m\, \partial f_1/\partial \mathbf{v}$: convective variation in velocity space
(action of external forces on the distribution function)

$\partial f_1/\partial t$ influence of the other particles
(1 denotes single particle distribution)

External Forces (can be any type of force):

$\mathbf{F} = q\,\mathbf{E}$ electric field
$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$ magnetic field

No influence of any other particles:

$$\partial f_1/\partial t + \mathbf{v}\, \partial f_1/\partial \mathbf{r} + \mathbf{F}/m\, \partial f_1/\partial \mathbf{v} = 0$$

Influence of collisions:

$$\partial f_1/\partial t + \mathbf{v}\, \partial f_1/\partial \mathbf{r} + \mathbf{F}/m\, \partial f_1/\partial \mathbf{v} = \partial f_1/\partial t|_{coll} = (f_1 - f_{10})/\tau$$

The right hand side has been expressed in the simplest approximation by the change through collisions between 2 particles (1, 2) with an average frequency $\nu_{12} = 1/\tau$.

Boltzmann Equation and Vlasow Equation

Let us split of individual (collisions) and collective forces (field based on charge distribution). We can add the collective forces to external forces:

Vlasow Equation:

$$\partial f_1/\partial t + v\, \partial f_1/\partial \mathbf{r} + (\mathbf{F}_{ext.} + \mathbf{F}_{int.})/m\, \partial f_1/\partial \mathbf{v} = 0$$

External Forces $\mathbf{F}_{ext}$:

$\mathbf{F} = q\,\mathbf{E}$ electric field
$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$ magnetic field

$\mathbf{F}_{int.} = q\mathbf{E}_{int} = q \int \sigma/\varepsilon_o\, dr = q \int e \int f(r,v)\, dv\, /\varepsilon_o\, dr$ **use f_1**

Now we have a self-consistent description, but with a nasty Integro-Differential Equation.

It can be solved together with Maxwell's equations
or for an electrostatic case just with Poisson's Equation

-> kinetic description of plasmas

necessary, if $v \approx v_{therm}$

If $v >> v_{therm}$ -> cold plasma approach

or if $v << v_{therm}$ -> warm plasma approach

We use only bulk properties of the plasma, not distribution!

$\mathbf{v}$ = bulk velocity
n = density
kT = mean kinetic energy

Collective Effects

Important length scales:

Landau length:

l_L: $kT = e\ q/(\varepsilon_o\ r)$ $l_L = r = eq/(\varepsilon_o\ kT)$

distance where individual force becomes dominant
q: may be multiply charged

Debye length:

$\lambda_D = \sqrt{kT/(\varepsilon_o\ n\ e^2)}$

distance to which thermal particles can create a disturbance (or shield the potential of 1 particle)

d: average distance of particles

in a plasma usually

$l_L \ll d \ll \lambda_D$

i.e. many particles in a Debye sphere, the collective field is much more important than collisions

-> plasma regimes

Macroscopic Parameters

Use: with:

$$\partial f_1/\partial t + \mathbf{v}\, \partial f_1/\partial \mathbf{r} + \mathbf{F}/m\, \partial f_1/\partial \mathbf{v} = (\partial f/\partial t)_{interaction} = 0$$

Let us assume a general function A(**r**,**v**,t) (with v^0, v^1, v^2 ..)

We calculate the mean of the quantity:

$$\overline{A(\mathbf{r},t)} = \int A(\mathbf{r},\mathbf{v},t)\, f\, d\mathbf{v} \,/ \int f\, dv = 1/n * \int A(r,v,t)\, f\, dv$$

and insert this into the Boltzmann equation. Then we get eqns. For the mean A (separately) for individual species j:

$$\partial/\partial t\, (nA) - n\, \partial A/\partial t$$

$$+ \Sigma\, \partial/\partial x_j\, (n\, v_j A) - \Sigma\, n\, v_j\, \partial A/\partial x_j$$

$$- n/m\, \Sigma\, F_j \partial A/\partial v_j = 0$$

I) $A = 1 = v^0$ $\qquad$ $A = 1$

II) $A = m\mathbf{v}^1$ $\qquad$ $A = m\mathbf{v} = m\mathbf{v}_0$

I) Conservation of particles (A = 1): Continuity Equation

$$\partial/\partial t\, n + \Sigma\, \partial/\partial x_j\, (n\, v_j) = 0$$

$$\partial/\partial t\, n + \nabla\, (n\, \mathbf{v}_0) = 0$$

Macroscopic Parameters

Averaged parameters:

$\partial/\partial t\ (nA) - n\ \partial A/\partial t$

$+ \Sigma\ \partial/\partial x_j\ (n\ v_j A) - \Sigma\ n\ v_j\ \partial A/\partial x_j$

$- n/m\ \Sigma\ F_j \partial A/\partial v_j = 0$

I) $A = 1 = v^0$ $\qquad$ $A = 1$

II) $A = m\mathbf{v}^1$ $\qquad$ $A = m\mathbf{v} = m\mathbf{v}_0$

II) Conservation of momentum ($A = m\mathbf{v}$):
Momentum Transport Equation

$$\partial/\partial t\ (n\ m\mathbf{v}_0) + \Sigma\ \partial/\partial x_j\ (n\ v_j\ m\mathbf{v}) - n/m\ \Sigma\ F_j \partial\ m\mathbf{v}_0/\partial v_j = 0$$

$$\partial/\partial t\ (n\ m\mathbf{v}_0) + \nabla\ (n\ \mathbf{v}\ m\mathbf{v}) - n\ \mathbf{F} = 0$$

After rearrangement with $\mathbf{v} = \mathbf{v}_0 + \mathbf{u}$
where $\mathbf{u}$ is the random component of $\mathbf{v}$

$$n\ m\ \partial/\partial t\ \mathbf{v}_0 + n\ m\ \mathbf{v}_0 \nabla\ \mathbf{v}_0 = - \underbrace{\nabla \cdot (n\ m\ \mathbf{u} \circ \mathbf{u})}_{\nabla \cdot P} + n\ \mathbf{F}$$

where P is the stress tensor. It contains the **pressure** p (diagonal terms) and **viscosity** ν (off diagonal). Let's stay with just the pressure. Closure of the problem:

$p = 0$: cold model $\qquad$ or hot model:

p determined by Equation of State: $P = n\ KT/\alpha = nm\ c_s^2$

One- and Two-Fluid MHD Equations

Chapters on MHD in Chen and Jackson
***Kallenrode*, pp.30-44; Kirk, pp. 12 – 24; Kivelson&Russell, p.41-50**

Now we have 2 sets of equations (for electrons and ions, species j)

$$\frac{\partial \rho_j}{\partial \mathbf{t}} + \nabla(\rho_j \mathbf{u}_j) = 0$$

I.

$$\rho_j \frac{\partial \mathbf{u_j}}{\partial \mathbf{t}} + \rho_j \mathbf{u}_j \nabla \mathbf{u}_j = - \nabla p_j + q_j \frac{\rho_j}{m_j}[\mathbf{E} + \mathbf{u}_j x \mathbf{B}] - \mathbf{F}_{other}$$

II.

and **Maxwell's Equations.** The **Lorentz force** has been used to represent external electromagnetic forces. Additional forces are summarized in $\mathbf{F}_{other}$. Invoking an **Equation of State** for p leads to a closed set for MHD problems. This represents a 2-fluid model of the plasma, which is very useful for the discussion of plasma waves and instabilities with significant influence from both species.

In our case this is still too complicated. We can basically treat the solar wind plasma as 1 fluid with some modifications. Adding both particle conservation equations leads to mass conservation, as already used before (I.3_1). Adding both momentum transport equations leads to

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \nabla \mathbf{u} = - \nabla p + \mathbf{j} x \mathbf{B} - \mathbf{F}_{other}$$

For the stationary case with gravity this is an equation similar to (I.3_2), but with an additional term for the magnetic field:

$$\rho \mathbf{u} \nabla \mathbf{u} = - \nabla p + \mathbf{j} x \mathbf{B} - \rho \frac{G M_s}{r^2} \hat{\mathbf{r}} \qquad (I.4\_1)$$

Here $\rho = m_i n_i + n_e m_e \approx m_i n_i$

$\mathbf{j} = en(\mathbf{u}_i - \mathbf{u}_e)$ $\quad$ $\sigma = e\,(n_i - n_e) \approx 0$ $\quad$ (because $q_i = -q_e$ and $n_i \approx n_e$)

The momentum equation does not contain **E** anymore, because any electric field will be shielded beyond the Debye length by the mobility of the electrons. Therefore, the Poisson equation will not be needed to determine E within our framework. This is called the "Plasma Approximation": Don't use the Poisson Equation in plasmas unless you absolutely have to. This applies for MHD problems! For certain small scale processes (waves, instabilities etc.) the Poisson Eqn. is needed again. **u** is the bulk flow of the combined mass density, which is pretty much the ion flow. The electron bulk flow adds little. We can neglect terms of order m_e/m_i.

The definition of **j** suggests another simplification in Maxwell's Equations. Let us do a dimensional analysis: $u \approx L/\tau$ where L and τ are typical spatial and temporal scales of the variations, which can be used for the spatial and temporal derivatives in an estimate ($\nabla \approx 1/L$; $\partial/\partial t \approx 1/\tau$).

In Faraday's Law: $E/L + B/\tau \approx 0$

Now let us compare the typical values for the displacement current with the left hand side in Ampere's Law:

$$\frac{1/c^2\, E/\tau}{B/L} \approx \frac{1/c^2\, BL/\tau^2}{B/L} \approx \frac{L^2}{c^2\tau^2} \approx \frac{u^2}{c^2} << 1 \qquad \text{(I.4\_2)}$$

I.e. for non-relativistic problems the displacement current can be neglected, and we only keep the current term $\mathbf{j}$.

The current $\mathbf{j}$ represents basically the motion of electrons w.r.t. ions in this fluid. This brings back the electric field solely for the motion of the electrons in the form of a "generalized Ohm's Law". We need the electron momentum transport equation (first without collisions). As we have stated before the electrons closely track the motion of the ions on a large scale ($L > \lambda_D$), i.e. the left hand side has already been taken care of in the momentum transport equation of the plasma flow (including any forces on the mass density, such as gravity). In other words: transformed into the rest frame of the mass flow, we are left with:

$$0 = -\nabla p_e - en\,(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) \qquad \text{(I.4\_3)}$$

With J: $\mathbf{u}_e = \mathbf{u}_i - \mathbf{j}/ne \approx \mathbf{u} - \mathbf{j}/ne$

$$0 = -\nabla p_e - en\,(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathbf{j} \times \mathbf{B}$$

We arrive at the same result deriving the generalized Ohm's Law by subtracting the two momentum equations (II) from each other after multiplying the electron eqn. by m_i and the ion eqn. by m_e. Terms of order m_e/m_i are neglected. Also for $u/c << 1$ the quadratic terms in u (convective derivatives) can be neglected for the relative motion of e and i. (Work through pp. 41 – 44 in Kallenrode for the step by step approach.) The end result (including collisions) is:

$$(m_e/e)\partial\mathbf{j}/\partial t = en\,(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \mathbf{j} \times \mathbf{B} + \nabla p_e - en\mathbf{j}/\sigma$$

where σ is the conductivity here! Note: c does not appear in the equation, if the MKSA system is used! (As I used for Maxell's Equations) I.4_3 follows from this more general equation, if we neglect electron inertia (left hand side, is neglected unless acceleration of electrons is treated in the problem) and have no collisions (last term).

Then we can rewrite the equation as follows:

$$(\mathbf{E} + \mathbf{u} \times \mathbf{B}) = \mathbf{j} \times \mathbf{B}/ne - \nabla p_e/ne \qquad \text{(I.4\_4)}$$

The terms on the R.H.S. of the Eqn. are of the order of the bulk flow parameters in I.4_1. We look at the solar wind at 1 AU, i.e. the gravitational term is negligible. With a similar dimensional argument as above this term $(\rho\mathbf{u}\nabla\mathbf{u})/ne$ becomes approximately:

$$\rho u^2/(neL) \approx \rho u/(ne\tau)$$

whereas the 2nd term on the L.H.S. is of order uB. Let us compare these two terms:

$$uB/(\rho u/ne\tau) \approx eB/m_i * \tau \approx \Omega_{ci} * \tau$$

The gyro frequency Ω_{ci} is about 0.5 s^{-1} in the solar wind, very short compared with relevant time scales, i.e. the time it takes the solar wind to pass a substantial distance. Thus the terms on the R.H.S. are unimportant. The Hall term **j x B** that produces an additional electric field perpendicular to B, which depends on the different mobility of the charge carriers, can be neglected here, as can the electron pressure gradient. However, these terms become important in laboratory situations with much smaller length scales and shorter time scales as well as for waves and disturbances in the solar wind.

Without collisions (we still neglected them so far!) we have:

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0 \qquad \text{(I.4\_5)}$$

or on long space and time scales electrons are so mobile that they short out any electric field **E'** in the moving system! The last point is important here: Since we first treated the mass flow in I.4_1 and then the electron motion relative to the bulk flow we have assumed that we are moving with the plasma. In this rest frame any electric field would lead to an infinite current. However, in the rest frame of any spacecraft from which we want to measure the parameters, the electric field is:

$$\mathbf{E} = - \mathbf{u} \times \mathbf{B}$$

We will make use of this relation again later in the course, when we will see that sometimes a pure mechanical treatment can be as appropriate as computation of trajectories in an electric field, if we just transform the problem into the correct frame of reference!! This will make several acceleration problems much simpler.

Now let us bring in collisions heuristically into the electron and ion momentum equations. This produces the additional terms:

$$\begin{array}{ll} \text{electrons} \ldots & + \rho_e \nu_{ei} (\mathbf{u}_i - \mathbf{u}_e) \\ \text{ions} \ldots & - \rho_e \nu_{ei} (\mathbf{u}_i - \mathbf{u}_e) \end{array} \qquad \text{(I.4\_6)}$$

Any collisions among identical species are unimportant, because they don't change the momentum of the species. The collisions between i and e act similarly on both species, i.e. there is no change in the total momentum equation, but through the electron momentum transport Ohm's Law gains an additional term, which in fact is the most familiar one:

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \rho_e \nu_{ei} (\mathbf{u}_i - \mathbf{u}_e)/ne = m_e \nu_{ei}\, en(\mathbf{u}_i - \mathbf{u}_e)/ne^2$$

with the conductivity: $\sigma = ne^2/m_e\nu_{ei}$ this becomes:

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{j}/\sigma \qquad \text{or} \qquad \mathbf{E} = - \mathbf{u} \times \mathbf{B} + \mathbf{j}/\sigma \qquad \text{(I.4\_7)}$$

Frozen-in Magnetic Field and Diffusion

Inserting this electric field into Faraday's Law:

$$\partial \mathbf{B}/\partial t = \nabla x(\mathbf{u} \ x \ \mathbf{B}) - \nabla \ x \ \mathbf{j}/\sigma$$

With Ampere's Law:

$$\partial \mathbf{B}/\partial t = \nabla x(\mathbf{u} \ x \ \mathbf{B}) - \nabla \ x \ (\nabla \ x \ \mathbf{B})/\sigma\mu_o \qquad \text{(I.4\_8)}$$

$$\nabla \ x \ (\nabla \ x \ \mathbf{B}) = \nabla(\nabla\mathbf{B}) - \nabla^2\mathbf{B} = -\nabla^2\mathbf{B} \qquad \text{(because } \nabla\mathbf{B} = 0\text{)}$$

$$\partial \mathbf{B}/\partial t = \nabla x(\mathbf{u} \ x \ \mathbf{B}) + \nabla^2\mathbf{B}/\sigma\mu_o$$

The first term on the R.H.S. represents the frozen-in magnetic flux.
The second term on the R.H.S. represents diffusion of magnetic flux.

$\eta = 1/\sigma\mu_o$ is the magnetic diffusivity

which reaches very small values in the corona and the solar wind, because the conductivity is so high. It is determined by the Coulomb collisions. Analogous to pure hydrodynamic flow we can define a Magnetic Reynolds Number (dimensionless):

$$R_m = Lu/\eta$$

where L is the typical scale length. It compares the "persistence" (Lu) of structures of size L in a flow with speed u with diffusivity η. Diffusivity has the known dimensions of a diffusion coefficient (m^2/sec).
For the solar wind this leads to incredibly large numbers of $\approx 10^{18}$. I.e. the solar wind and the magnetic field are well coupled to the flow and don't mix easily. The diffusion term can be neglected in most problems. To estimate the strength of the diffusion term we only have to consider the most important collisions, i.e. what yields the highest collision frequency at given location. (Refer to Homework Set #2.)

Magnetic Field in a Flux Tube

Let us consider a flux tube. The rate of the change of the flux is:

$$\frac{d}{dt}\int \boldsymbol{B}d\boldsymbol{S} = \int_s \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{S} + \int_c \boldsymbol{B}(\boldsymbol{u}x d\boldsymbol{l})$$

i.e. the explicit change of B in the flux tube and the change of the area S of the tube because the line element d**l** of the boundary curve C sweeps out the area **u**dt x d**l** during the motion of the flux tube with the plasma.

$\mathbf{B}(\mathbf{u} \ x \ d\mathbf{l}) = -(\mathbf{u} \ x \ \mathbf{B})d\mathbf{l}$ and Stokes' Theorem lead to:

$$\frac{d}{dt}\int \mathbf{B}d\mathbf{S} = \int_s [\frac{\partial \mathbf{B}}{\partial t} - \nabla \mathbf{x}(\mathbf{u} \ \mathbf{x} \ \mathbf{B})]d\mathbf{S} \qquad \text{(I.4\_9)}$$

which vanishes for frozen-in magnetic fields. This means that the magnetic flux in a specific flux tube will be conserved in the solar wind, no matter how the flux tube changes shape or size on its way. Also in the case of a flow parallel to B the flux has to follow any widening or compression to leading order. The magnetic field acts like a "physical" tube.

$\partial B/\partial t = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla^2\mathbf{B}/\sigma\mu_o$ (i.e. including collisions) is the basis of the dynamo theory

$\nabla \times (\mathbf{u} \times \mathbf{B})$ "creates magnetic field" via convective motion whereas

$\nabla^2\mathbf{B}/\sigma\mu_o$ dissipates field

Magnetic Pressure and Tension

The Lorentz term in the momentum transport equation turns into a double cross product (after using Maxwell's Eqns.). It can then be written:

$$\mathbf{F} = \mathbf{j} \times \mathbf{B} = (\nabla \times \mathbf{B}) \times B/\mu = (\mathbf{B}\nabla)\mathbf{B}/\mu - \nabla(B^2/2\mu) \qquad \text{(I.4\_10)}$$

which describes the magnetic force on the plasma in the flux tube.

The 2nd term is similar to the pressure gradient ∇P, i.e. like a magnetic pressure, which acts in the direction of the change of the magnetic field, from higher to lower magnetic pressures (along the field). The first term represents the tension of curved magnetic field lines (a homogeneous field would produce neither of the terms). $\mathbf{F}\,\mathbf{B} = 0$ as can be seen from the original equation. The force is strictly perpendicular to B (because of the cross product). In a gradient a component in the direction of the flux tube emerges that is similar to the force with which we squeeze something out of a tube.

It is interesting to note that the magnetic tension is twice as high (2 directions!!) as the magnetic pressure (1 direction!!).

Analogous to the particle pressure ($c_s^2 = \gamma p/\rho$):

$$v_A^2 = B^2/\mu\rho$$

acts like a sound speed (if we interpret the term on the R.H.S. as magnetic pressure). v_A is the Alfvén speed, propagation similar to waves on a string (rubber band). We can imagine magnetic field lines like a rubber band.

Transport of Magnetic Field by the Solar Wind

Kirk, pp. 52 - 54

Depending on which is stronger the wind or the magnetic field, the magnetic field is dragged along or the wind will be trapped. The latter seems to happen in the helmets of the corona. Otherwise the wind takes solar magnetic field along.

Let us consider a solar wind element which is moving outward. It takes magnetic field along radially with the solar wind speed v_{sw}. Since the sun is rotating with a period of = 28 days at the equator, the field lines are spiraled around. They look like the water emerging from a rotating garden hose.

Viewgraph

However, the magnetic field is forced to rotate with the sun with the speed

$$v_t = \Omega(r - R_s)$$

This leads to an angle Ψ:

$$\tan\Psi = \Omega(r - R_s)/v_{sw} \qquad \text{(I.4\_11)}$$

Typical angles with respect to the flow velocity at the Earth are ≈ 45°

Effect of the Magnetic Field on the Solar Wind
(Weber - Davis problem) **Kirk, pp. 54 - 56**

After the discussions above there must also be an effect of B on the solar wind. The magnetic field remains attached to the rotating sun, while the solar wind drags it (frozen-in!) radially outward. This should lead to a transport of angular momentum, as one can imagine with magnetic field lines pictured like rubber bands. (Let us treat the problem restricted to the equatorial plane, i.e. no latitude effects. We are interested in effects in the azimuthal direction.):

$$\mathbf{B} = (B_r(r), 0, B_\Phi(r)) \quad \text{and} \quad \mathbf{u} = (u_r(r), 0, u_\Phi(r))$$

Because the wind is still assumed isotropic

$$\nabla\mathbf{B} = 0 \qquad \text{leads to } r^{-2}\partial/\partial r(r^2 B_r) = 0$$

$$B_r(r) = B_o R_s^2/r^2 \qquad \text{(I.4\_12)}$$

i.e.: $B_r r^2$ = constant

Incorporating the magnetic field into our original equation set (I.3_1 - I.3_3)

I.) $$\rho(\mathbf{u}\,\nabla)\mathbf{u} = -\nabla\left(P + \frac{B^2}{2\mu}\right) + \left(\frac{\mathbf{B}}{\mu}\nabla\right)\mathbf{B} - \rho\frac{GM_s}{r^2}\hat{\mathbf{r}} \qquad \text{(I.4\_13)}$$

II.) $$\nabla(\rho\mathbf{u}) = 0 \qquad \text{(I.4\_14)}$$

like I.4_12: $\rho u_r\, r^2$ = constant (through integration)

III.) polytropic eqn. of state

$$P/r^\alpha = \text{constant} \qquad \text{with } P = \rho kT/m \qquad \text{(I.4\_15)}$$

and: $$\nabla \times (\mathbf{u} \times \mathbf{B}) = 0 \qquad \text{(I.4\_16)}$$

because collisionless and stationary ($\partial\mathbf{B}/\partial t = 0$)

The latter equation leads to

$$r(u_r B_\Phi - u_\Phi B_r) = \text{constant}$$

Φ component: $\nabla \times (\mathbf{u} \times \mathbf{B})|_\phi = 1/r\ \partial/\partial r(r(u_r B_\Phi - u_\Phi B_r)) = 0$ _

With $B_\Phi = 0$ and $u_\Phi = \Omega R_S$ at the solar surface this means:

$$r(u_r B_\Phi - u_\Phi B_r) = -\Omega R_S^2 B_o = -\Omega r^2 B_r \quad \text{or}$$

$$B_\Phi = (u_\Phi B_r - \Omega r B_r)/u_r \qquad \text{(I.4\_16a)}$$

The only terms with a Φ component in (I.4_13) are:

$$\rho(\mathbf{u}\nabla)u_\Phi = (\mathbf{B}\nabla)B_\Phi/\mu_o$$

With azimuthal symmetry:

$$\mu_o \rho u_r\, d/dr(r u_\Phi) = B_r\ d/dr(r B_\Phi)$$

with $\rho u_r/B_r = \text{constant}$ from (I.4_12 and _14)

As a result we get the conservation of the angular momentum L:

$$r u_\Phi - (B_r/(\mu_o \rho u_r)) \cdot r B_\Phi = L \qquad \text{(I.4\_17)}$$

L is the angular momentum per unit mass carried outward with the solar wind. This transmits a torque force on the solar surface, which has slowed down the solar rotation in the past and is still going on. This may have solved part of the angular momentum problem of the sun from formation until today. The sun has much less angular momentum than the original interstellar gas cloud out of which the solar system formed. (Homework Problem)

After substituting I.4_16a into I.4_17 another critical point solution emerges:

$$\boldsymbol{u_\varphi} = \Omega \boldsymbol{r} \frac{M_{\boldsymbol{A}}^2 L/(\boldsymbol{r}^2\Omega) - 1}{M_{\boldsymbol{A}}^2 - 1} \quad \text{with } M_{\boldsymbol{A}}^2 = \boldsymbol{\mu_o \rho u_r^2 / B_r^2} \qquad \text{(I.4\_18)}$$

as the Alfvénic Mach number. The critical point is reached at $r = r_A$. This implies that $L = \Omega r_A^2$, i.e. the solar wind co-rotates with the sun as a rigid rotator up to the Alfvén point.

The radial component of the momentum transport can also be brought into a form with a critical point. In this case the magnetic pressure and the thermal pressure on the R.H.S. are combined, which leads to a magnetic sound speed close to the Alfvén speed. Therefore, 2 new critical points are introduced by the addition of the magnetic field, which are close together.

Viewgraph

Let us estimate r_A from the fact that at r_A

$$1 = M_A^2 = \frac{\mu_o \rho u_r^2}{B_r^2}$$

With $B_r \bullet r^2 = \text{const.} = B_r(r_A) \bullet r_A^2$

and $\rho\ u_r\ r^2 = \text{const.} = \rho(r_A)\ u_r(r_A) \bullet r_A^2$

this leads to

$$r_A = r \frac{B_r(r)}{\sqrt{\mu_o \rho(r) u_r(r_A)/u_r(r)} \cdot u_r(r)}$$

We can express r_A in terms of values for the solar wind parameters at 1 AU (with $B_r \approx 5$ nT typically) except for the need for $u_r(r_A)$. This could in principle be solved by using $u_r(T)$ from Parker's solution, but is tedious. For our estimate it may suffice that $u_r(r_A)$ has to be higher than c_s and lower than $u_r(r_E)$ from the position of the Alfvén point. Take the stand that roughly $u_r(r_A) \approx (u_r(r_E) + c_s)/2$ (as an experimenter's first guess). This leads to a rough estimate r_A for the determination of L in Homework #2.

Azimuthal Structure of the Solar Wind

Isenberg, pp. 1-15

It is observed that the sign of the radial component B_r changes quasiperiodically.

Viewgraph

The solar wind is sectored. This is due to the fact that we receive solar wind either from the northern half or southern half (magnetically) of the sun.

Viewgraph

The boundary is warped like the skirt of a ballerina. This is called the "ballerina model" of the interplanetary magnetic field. As the solar wind streams freely from the N and S coronal holes, while it is impeded over the equatorial region, we now expect an effect on the solar flow. This is indeed seen.

Viewgraph

At the sector boundaries the wind is slow!!

What, if we look at the wind at high latitudes? This is what ULYSSES was made for!

Viewgraph

At low latitudes we still see the sector structure, but beyond ≈40° this is gone and a fast wind persists. Beyond 40° the average speed is still increasing slowly.

Viewgraph

I.e. the more typical fast solar wind has a very steady speed of

$$v_{sw} \approx 800 \text{ km/s}$$

while the slow wind in the equatorial regions is highly variable and is much slower (250 – 500 km/sec). The fast wind originates in the coronal holes with open magnetic field lines. The slow wind is thought to originate in the closed field regions over the equator, but this is not completely clear yet. Models of an origin in the boundary regions between open and closed field are also discussed. In general, the acceleration of the wind, both fast and slow, is not well understood yet.

The boundaries between the two types of wind are very sharp. This can be seen very clearly in the solar wind composition, an abrupt change with not much mixing. What we see in the ecliptic, is determined by the magnetic field interaction and the chasing of the fast and slow wind streams that are associated with these boundaries. For example, fast wind runs away from slow wind or vice versa.

Viewgraph

This causes compressions and rarefactions. Where fast wind runs into slow wind, the density increases, followed by a significant reduction.

Viewgraph

These velocity variations cause strong disturbances in the wind. The individual streams don't mingle, so they are obstacles to each other. This is true also of planetary magnetospheres. They represent substantial obstacles to the solar wind. In all these cases the wind that is intimately connected with the magnetic field (as discussed earlier) gets stopped or disturbed. The magnetic field serves as a mediator of these disturbances on the solar wind. On the other hand, magnetic field disturbances, which contain (magnetic) energy, can heat or accelerate plasma, because they are forced to move through the plasma. This may happen in the solar corona and in the SW acceleration region to provide the missing energy.

To understand the basics, let us start to work on hydromagnetic waves in plasmas. We will need them also for the particle acceleration later on.

I.5 Hydrodynamic Waves

Kallenrode, p. 67 - 75
Kivelson&Russell, p. 330-342; Kirk, pp. 37-42

Let us now waves in plasmas. This is an important aspect in any plasma and applies in many ways to the solar wind. To create a wave **a medium that can move around** and **a restoring forc**e are needed. In plasmas many different restoring forces, such as electric, magnetic and pressure are active, and the different species may react in different ways. For the solar wind applications we will restrict ourselves to waves that deal with macroscopic motions.

Let us consider for the beginning homogeneous plasmas at rest without **B**. Such a plasma is not much different from a neutral gas, except that two different species must be considered. Therefore, we expect **sound waves** in plasmas.

Sound Waves

For the analysis we use average parameters of the plasma A_o and small variations around the average A_1 for mass density, velocity and pressure. Remember that the plasma is at rest, i.e. $u_o = 0$:

$$\rho = \rho_o + \rho_1 \qquad \vec{u} = \vec{u}_1 \qquad p = p_o + p_1$$

Then we start with the MHD equations:

I) $$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{u}) = 0$$ Mass conservation

II) $$\rho\frac{d\vec{u}}{dt} = -\nabla p$$ Momentum Equation (I.5_1)

III) $$\frac{P}{\rho^\alpha} = const. = \frac{P_o}{\rho_o^\alpha}$$ Equation of State

The backgound plasma is assumed stable and in equilibrium. Therefore, we only need to consider variations of the small disturbances ρ_1, $\vec{u}_1$, p_1

Because we use only small perturbations ($A_1 << A_o$), all terms of order ρ_1^2 etc. are omitted.

II a) $$\rho_o\frac{\partial \vec{u}_1}{\partial t} = -\nabla p_1 = -c_s^2\nabla\rho_1 \qquad (I.5\_2)$$

using: $p_1 = c_s^2\rho_1$ with $c_s^2 = \dfrac{\alpha P_o}{\rho_o} = \text{const.!!!}$

Apply $\left.\dfrac{\partial}{\partial t}\right|$ Ib) $$\frac{\partial \rho_1}{\partial t} + \rho_o\nabla\vec{u}_1 = 0$$ note: $u_o = 0!$

After differentiating Ib) again for time and inserting IIa), we get:

$$\frac{\partial^2 \rho_1}{\partial t^2} - c_s^2 \Delta \rho_1 = 0 \qquad \text{or} \qquad \frac{\partial^2 \rho_1}{\partial \boldsymbol{t}^2} - \boldsymbol{c}_{\boldsymbol{s}}^2 \nabla^2 \rho_1 = 0 \qquad \text{(I.5\_3)}$$

($\Delta = \nabla^2$ in the European notation) This is a wave equation.

With the Ansatz: $\rho_1 = \rho_{10} \bullet \rho^{i(\vec{k}\vec{r} - \omega t)}$ we get

$$\omega^2 \rho_{10} - c_s^2 k^2 \rho_{10} = 0$$

or the dispersion relation:

$$\omega^2 = c_s^2 k^2 \qquad \text{(I.5\_3a)}$$

for ordinary sound. It describes isotropic propagation in a plasma.

(I.5_2a) becomes

$$\rho \omega \vec{u}_1 = -\vec{k} c_s^2 \rho_1$$

The **velocity disturbance is parallel to the propagation**, i.e. we have a **longitudinal wave** or pressure wave.

This is the typical analysis (cooking recipe) for waves:
- assume a small disturbance
- use linear equations in ρ_1, u_1, p, etc.
- apply a wave Ansatz

<u>Alfvén Waves</u>

Let us now include the magnetic field into our considerations. The simplest case is assuming a purely magnetic restoring force that moves around the plasma material. If we imagine magnetic field lines as rubber bands, we can expect by analogy a wave propagating along the string with:

$$v_A^2 = \frac{B^2}{\mu_o \rho}$$

To derive this from scratch, let us add $\vec{\boldsymbol{B}}_{\boldsymbol{0}} + \vec{\boldsymbol{B}}_1$ (or $\vec{\boldsymbol{j}}_1$ instead), but start with $p_o = 0$, i.e. with a cold plasma approximation.

I) $$\frac{\partial \rho_1}{\partial t} + \nabla(\rho_o \vec{u}_1) = 0 \qquad \text{(I.5\_4)}$$

II) $$\rho_o \frac{\partial \vec{u}_1}{\partial t} = \vec{j}_1 \, x \, \vec{B}_o = (\nabla \, x \, \vec{B}_1) \, x \, \frac{\vec{B}_o}{\mu_o}$$

In addition, from Maxwell's Equations:

III) $$\frac{\partial \vec{B}_1}{\partial t} = \nabla \, x \, (\vec{u}_1 \, x \, \vec{B}_o)$$

in a collisionless plasma with $\vec{u}_1 << c$ and t >> $1/\Omega_{ci}$

and $$\nabla \vec{B}_1 = 0$$

With a wave Ansatz($e^{i(\vec{k}\vec{r}-\omega t)}$), the following equations emerge:

I) $$-\omega\rho_1 + \rho_o \vec{k}\vec{u}_1 = 0$$

II) $$-\mu_o\rho_o \omega\vec{u}_1 = (\vec{k} \, x \, \vec{B}_1) \, x \, \vec{B}_o = \vec{B}_1(\vec{B}_o \bullet \vec{k}) - \vec{k}(\vec{B}_1\vec{B}_o)$$

III) $$-\omega \vec{B}_1 = \vec{k} \, x \, (\vec{u}_1 \, x \, \vec{B}_o) = (\vec{B}_o\vec{k})\vec{u}_1 - \vec{B}_o(\vec{k}\vec{u}_1)$$

and $\vec{k}\vec{B}_1 = 0$

We start with transverse waves (string waves) i.e. $\vec{k} \bullet \vec{u}_1 = 0 \quad \perp!$

From II) $\vec{k} : 0 = -\mu_o\rho_o \omega \vec{k}\vec{u}_1 = (\vec{k} \bullet \vec{B}_1)(\vec{B}_o \bullet \vec{k}) - k^2(\vec{B}_1 \bullet \vec{B}_o)$

Follows:

$$\vec{B}_1 \bullet \vec{B}_o = 0 \qquad \vec{B}_1 \perp \vec{B}_o$$

We get from Maxwell's Equations (III):

$$\vec{B}_1 = -\frac{1}{\omega}(\vec{B}_o\vec{k})\vec{u}_1$$

By inserting into II)

$$-\mu_o\rho_o \omega\vec{u}_1 = -\frac{1}{\omega}(\vec{B}_o\vec{k}) \bullet (\vec{B}_o \bullet \vec{k}) \bullet \vec{u}_1$$

We allow an arbitrary propagation $\vec{k}$ with respect to B_o at angle θ. Therefore:

$$\vec{B}_o\vec{k} = B_o k \cos\theta$$

This leads to the **dispersion relation**

$$\omega^2 = \frac{B_o^2}{\mu_o \rho_o}\cos^2\theta\, k^2 = V_A^2 k^2 \cos^2\theta \qquad \text{(I.5\_5)}$$

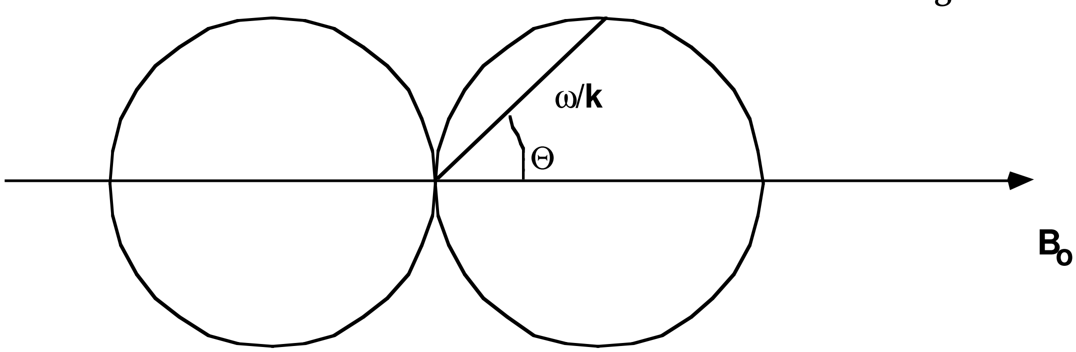
for Alfvén waves, where

$$V_A = \sqrt{\frac{B_o^2}{\mu_o \rho_o}} \qquad \text{Alfvén velocity}$$

While sound waves are longitudinal waves, Alfvén waves are pure transverse waves. The propagation velocity is based on the B-field component parallel to $\vec{k}$.

$$\frac{\omega}{k} = V_A \bullet \cos\theta$$

Phase diagram



By adding the magnetic diffusion term $\eta\nabla^2 \vec{B}_1$ to relation II) we get

IIb) $$-\mu_o \rho_o\, \omega \vec{u}_1 = \vec{B}_1(\vec{B}_o \bullet \vec{k}) + i\eta k^2 \vec{B}_1$$

In this way we simply add a damping term to the dispersion relation:

$$\omega^2 = \boldsymbol{V_A^2 k^2} \cos^2\theta - \boldsymbol{i}\omega\eta\boldsymbol{k}^2 \qquad \text{(I.5\_7)}$$

With the phase velocity of the wave

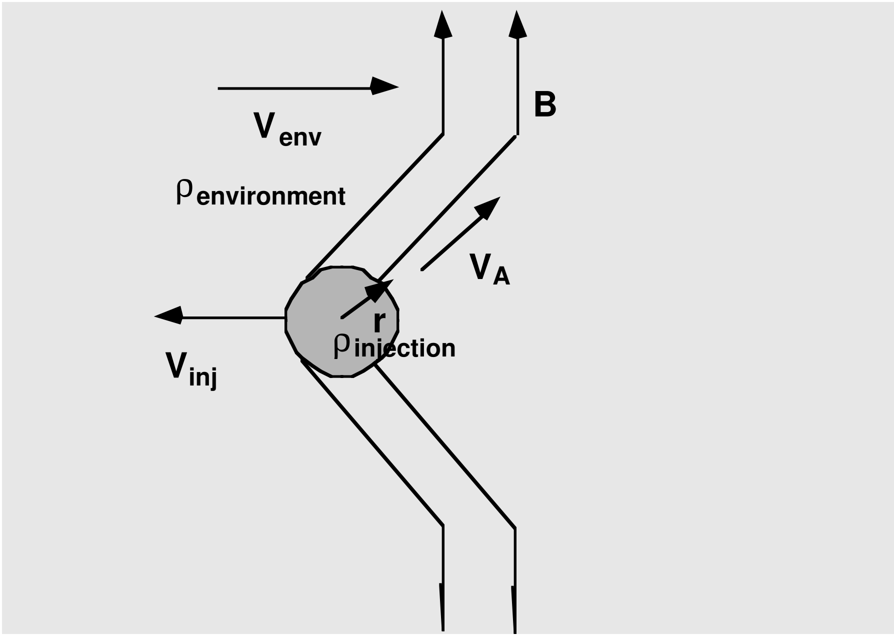
$$\frac{\omega}{k} = V_A \cos\theta$$

and the wavelength $\lambda = \frac{2\pi}{k}$ as the typical scale length we derive a magnetic Reynolds number $\boldsymbol{R_m} = \frac{\lambda \boldsymbol{V_{Ph}}}{\eta} = \frac{2\pi \boldsymbol{V_A} \cos\theta}{\boldsymbol{k}\eta}$.

$$\omega^2 = \boldsymbol{V_A^2 k^2} \cos^2\theta - \boldsymbol{i}\omega \frac{2\pi \boldsymbol{k}\, \boldsymbol{V_A} \cos\theta}{\boldsymbol{R_m}} \qquad \text{(I.5\_8)}$$

As for a damped oscillator, the real part of the frequency ω is reduced, and the wave will decay slowly. However, for the huge magnetic Reynolds numbers found in the solar wind, this effect can be neglected, except for very short wavelengths.

Let us discuss a simple example of a conductive obstacle in a magnetized plasma flow. Let there be a spherical cloud of plasma with radius r, and mass density ρ_{inj}.....



The magnetic field is frozen into the plasma flow. The newly injected cloud with ρ_{inj} is also attached to the magnetic field, i.e., frozen-in. Let us assume injection with v = 0. Thus the affected flux tube is stopped and forces the local environmental plasma to stop. However, the magnetic field transports that information further. 2 Alfvén waves are launched, which provide momentum coupling to additional environmental plasma in the corresponding flux tube. The affected volume is expanding at the speed V_A. We get for momentum conservation between the injected mass ($4\pi/3\ \rho_{inj} r^3$) and the accumulating mass ($2\ \rho_{inj}\pi r^2\ V_A$) with the average speed $(v_{env} - v)/2$.

$$\rho_{inj} \bullet \frac{4\pi}{3} \boldsymbol{r}^3 \frac{\boldsymbol{dv}}{\boldsymbol{dt}} = (\boldsymbol{v}_{env} - \boldsymbol{v})/2 \bullet \frac{\boldsymbol{dm}_{env}}{\boldsymbol{dt}} = 2\rho_{env} \bullet \pi \boldsymbol{r}^2 \bullet \boldsymbol{V}_A \bullet (\boldsymbol{v}_{env} - \boldsymbol{v})/2$$

$$\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \frac{\rho_{env}}{\rho_{inj}} \bullet \frac{3\boldsymbol{V}_A}{4\boldsymbol{r}} \bullet (\boldsymbol{v}_{env} - \boldsymbol{v})$$

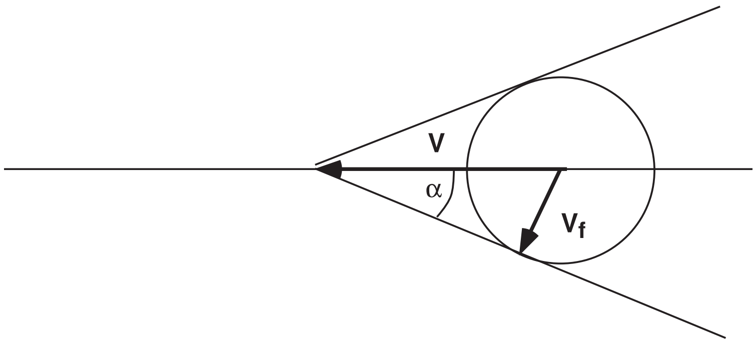
$$\textbf{\textit{or}} \qquad \boldsymbol{v} = \boldsymbol{v}_{env}\left(1 - \boldsymbol{e}^{-\frac{\rho_{env}}{\rho_{inj}} \frac{3\boldsymbol{V}_A}{4\boldsymbol{r}} \bullet \boldsymbol{t}}\right)$$

An object of small finite mass will be accelerated slowly in the beginning and then ultimately be swept away. The situation is reminiscent of comets, with the difference that they continuously dump material into the solar wind, as they are close to the sun.

This approach works as long as $v < V_A$ or the relevant wave speed, which can be up to $v_f = \sqrt{c_s^2 + V_A^2}$ perpendicular to B. v_f is the speed of a so-called fast mode wave (combined Alfvén and sound wave) in a plasma. The information about the disturbance cannot be sent fast enough into the environment; the waves pile-up and create a shock wave.

I.6. Shock Waves

The supersonic (super-Alfvénic) object is at the tip of a cone with the shock waves. The cone angle is determined by the Mach number of the object's speed.



$$\sin\alpha = \frac{v_f}{v} = M$$

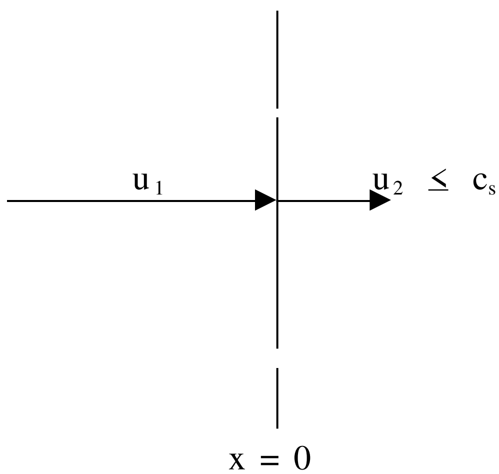
This is the relevant angle at distances large compared to the size of the object.
Near the object the shape of the shock wave is determined by the geometry of the flow around the object.

Let us concentrate on idealized shock geometry.

- plane
- perpendicular to the flow

Kallenrode, 119 – 130, Kirk, pp. 42-48
Jones, Ellison, Chapter 3.1

Let us first look at a pure hydrodynamic shock without magnetic field. In order to keep the shock at a fixed position the information about the flow obstacle has to reach the shock. The maximum signal speed in a gas is c_s (the sound speed). Therefore, the flow speed has to drop below the sound speed. This occurs at the shock.



We assume a jump instead of the finite transition.

Now we use again (for stationary flow)

I) $\dfrac{\partial(\rho\vec{u})}{\partial x}=0$ mass flux

II) $\dfrac{\partial}{\partial x}\left(\rho u^2+P\right)=0$ momentum flux

III) $\dfrac{\partial}{\partial x}\left(\dfrac{1}{2}\rho u^2\bullet\vec{u}+\dfrac{\gamma}{\gamma-1}P\vec{u}\right)=0$ energy flux conservation

For monotonic gas the overall energy ("Enthalpy") consists of

$$\frac{3}{2}P+P=\frac{5}{2}P$$

inner energy + work of expansion/compression

$$\frac{5}{3}=\gamma=\frac{C_p}{C_V}=\frac{f+2}{f}$$

where f is number of degrees of adiabatic coefficient γ (It is not the polytropic index α.).

At the shock (only x-components are considered, because perpendicular to the flow!):

I) $\rho_1 u_1 = \rho_2 \bullet u_2$

II) $\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$ (I.6_3)

III) $\left(\dfrac{\rho_1}{2}u_l^2+\dfrac{5}{2}P_1\right)u_1=\left(\dfrac{\rho^2}{2}u_2^2+\dfrac{5}{2}P_2\right)u_2$

(4.29 in Kirk et al.: $P_2 \bullet \underline{v_2}$! v_2 is missing)

From I) we get

$$\frac{\rho_2}{\rho_1}=\frac{u_1}{u_2}=r$$ the compression ratio at the shock

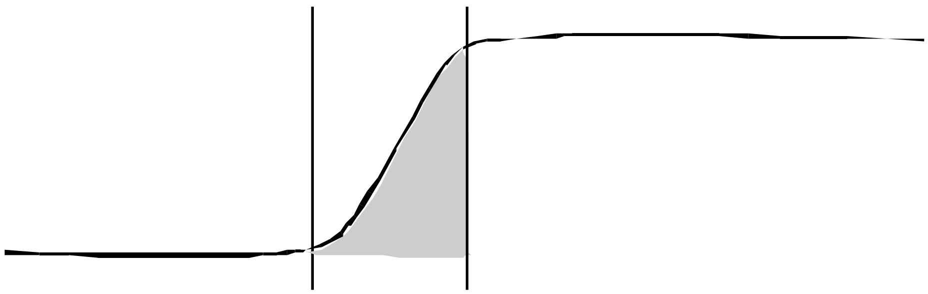
The set of equations has the trivial solution $[\quad]_1^2=0$, i.e. nothing changes.
With using $\rho u = \rho_1 u_1 = \text{const.}$

III) becomes:

$$\rho+u_1\frac{u_1^2}{2}+\frac{5}{2}P_1u_1=\rho_1u_1\frac{u_2^2}{2}+\frac{5}{2}P_2u_2$$

If we assume ρ_1, u_1, p_1 are given, there is only one other (non-trivial) solution for ρ_2, u_2, p_2, because we have 3 second order equations that determine the solution. No intermediate states are possible!

Therefore, the description of the transition itself requires the addition of higher order terms, i.e. explicit transport equations that contain diffusion, viscosity, etc. to get a finite smooth transition._



_We need to add higher order processes! For the time being we will ignore this.

The solutions for the equations are the so-called Rankine-Hugoniot Jump Conditions for the flow parameters. With

$$M_1^2 = \frac{\rho_1 u_1^2}{\gamma P_1} = \frac{u_1^2}{c_{s1}^2}$$

We get:

$$r = \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} \qquad \text{(I.6\_4)}$$

$$P_2 = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} P_1$$

(the latter one after some tedious algebra!!)

We don't want to do just algebra. Therefore, in HW#3, Problem 1), it is sufficient to write down the equivalent to I.6-3. The physical arguments for the terms are important!!

We can show for $M_1 \rightarrow \infty$.

$$1 \le r \le \frac{\gamma+1}{\gamma-1} = \frac{8/3}{2/3} = 4$$

whereas $\frac{P_2}{P_1} \sim M_1^2$ for large M_1 (~ means proportional to), i.e.:

$$\frac{P_2}{P_1} \sim \frac{\rho_2}{\rho_1}\frac{T_2}{T_1} \sim r\frac{T_2}{T_1} \qquad \text{(I.6\_5)}$$

There is strong heating in the shock. The reverse would be cooling and speeding up.
Therefore: $M_1 < 1$ does not lead to a shock solution.
Entropy would decrease!

$$S = c_v \log \frac{P}{\rho^\gamma}$$

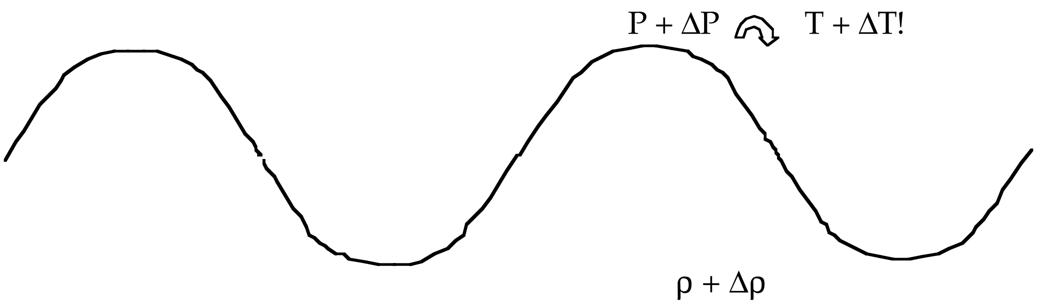
P increases with larger power than ρ^γ across the shock, no adiabatic process!

$M_1 > 1$

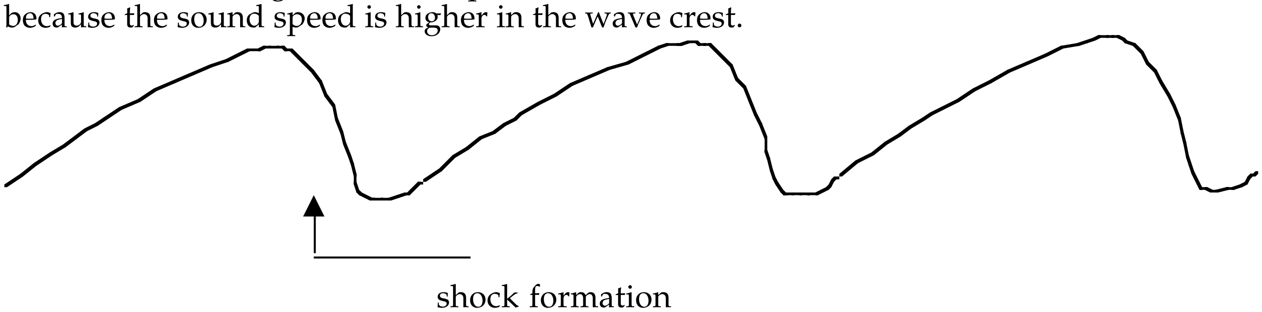
or: $u_2 < c_{s2}$ subsonic

<u>Why do we get a shock?</u>
Compression leads to heating. Since the pressure goes like:

$P \sim \rho^\gamma$ The temperature T must increase.



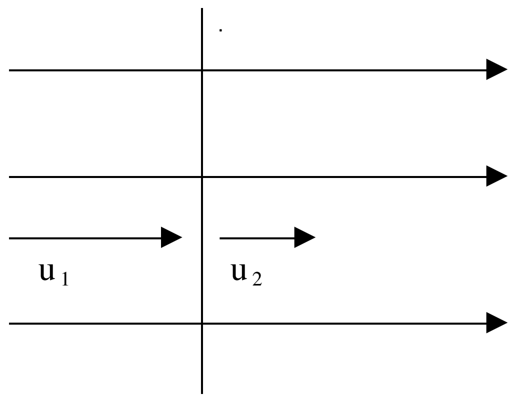
In a wave with longitudinal components (acoustic wave) the wave crests move faster, because the sound speed is higher in the wave crest.



This is a non-linear effect. Thus the linearization that we have used before would not be applicable anymore for shocks!

<u>With magnetic field</u>

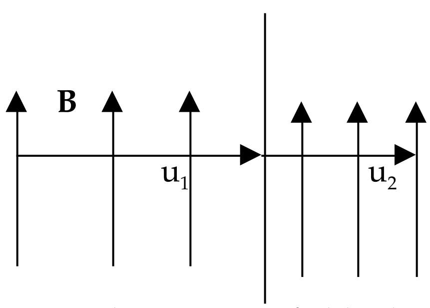
a) B|| to shock normal



We use the same equations as above and: $\nabla \vec{B} = 0$
The magnetic field is included only as “guard rails” for the flux tubes.

This is a Parallel Shock (or a quasi-parallel shock for small angles)
In this case the Alfvén wave does not steepen (no shock!); only the acoustic wave steepens to an acoustic shock.

b) B ⊥ to Shock Normal **Perpendicular Shock**



frozen-in
magnetic field

In this case the magnetic field piles up at the shock!
In other words, the magneto-acoustic modes steepen and therefore form a shock.

For the case perpendicular to B

I) $\rho_1 u_1 = \rho_2 u_2$ mass flow

II) $$\rho_1 u_1^2 + \underbrace{P_1 + \frac{B_1^2}{2\mu_o}} = \rho_2 u_2^2 + \underbrace{P_2 + \frac{B_2^2}{2\mu_o}}$$ momentum flow

We add the magnetic pressure to the equation. There is no magnetic tension involved, because there is no “kink” in the field.

III) $$\left(\frac{\rho_1 \boldsymbol{u}_1{}^2}{2}+\frac{\gamma}{\gamma-1}\boldsymbol{P}_1+\underbrace{\frac{\gamma_M}{\gamma_M-1}\frac{\boldsymbol{B}_1^2}{2\mu_o}}_{\boldsymbol{B}_1^2/\mu_o}\right)\boldsymbol{u}_1=\left(\frac{\rho_2 \boldsymbol{u}_2^2}{2}+\frac{\gamma}{\gamma-1}\boldsymbol{P}_2+\frac{\gamma_M}{\gamma_M-1}\frac{\boldsymbol{B}^2}{2\mu_o}\right)\boldsymbol{u}_2$$

energy flow

The equivalent expression for the magnetic field term is used: We have 2 degrees of freedom, i.e. f = 2 for magnetic field pressure, ||B and ⊥B. In other words the pressure of B contributes to the inner energy and to the work of compression.

Thus: $$\gamma_M=\frac{2+2}{2}=2$$

From Maxwell's Equations and Ohm's Law we use:

IV) $$\nabla x(\vec{u}\, x\, \vec{B})=0$$

$\Rightarrow$ u • B = const.

$\Rightarrow$ $B_1\, u_1 = B_2\, u_2$

I.e. $$r=\frac{\rho_2}{\rho_2}=\frac{u_1}{u_2}=\frac{B_2}{B_1} \qquad \text{(I.6\_7)}$$

$\Rightarrow$ $\dfrac{B_2^2}{B_1^2}=r^2$ for magnetic pressure!

Also in this case:

$$M_1\to\infty \quad B_2>\frac{\gamma+1}{\gamma-1}\bullet B_1$$

4 for $\gamma = 5/3$

But γ is different!

Because $$\boldsymbol{P}_{Total}=\boldsymbol{P}+\frac{\boldsymbol{B}^2}{2\mu_o}$$

$$u_1>v_f=\sqrt{c_{s1}^2+V_A^2} \qquad \text{(I.6\_8)}$$

The shock parameters are determined by the fast magneto-sonic Mach number: $M_f{}^2=\dfrac{\rho_1 u_1^2}{\gamma P_{Total1}}$. Because of the "mixture" in the total pressure, the effective γ_{eff} of the magnetized plasma will be

$$\frac{5}{3}=\gamma<\gamma_{eff}<\gamma_M=2$$

With

$$r=\frac{(\gamma+1)\,M^2}{(\gamma-1)\,M^2+2}$$

r will be reduced over the hydrodynamic shock. $M\to\infty$ $r\to 3$ for $\gamma_M = 2$, for example.

In a perpendicular shock

$$u_1 B_1 = u_2 B_2 \qquad \textit{leads to}$$

$$E_1 = E_2 \qquad (\textit{the } \vec{u} \times \vec{B} \textit{ field!})$$

This means, sitting at a stationary shock, one would feel the electric field. If the shock were able to keep a charged particle just in its plane, this particle would gain energy in the electric field.

In the case of a parallel shock there is no such field. We can work just with mechanics of particles, if they were just moving along the magnetic field lines. We use this idea to simplify the situation for the

<u>Oblique Shock</u>

The magnetic field has both | | and ⊥ components.

$$\vec{E}_1 = \vec{u}_1 \times \vec{B}_1$$

However, if we move parallel to the shock exactly with the speed of the magnetic field lines, there would be no electric field in this frame.

$$\vec{E}_1^{\,1} = 0$$

Viewgraph

If we look, for example, at a particle, which is reflected exactly normal to the shock, we have to consider the electric field at the shock. The problem becomes simpler, if we transform the problem into a system without an electric field E. Then we perform the particle reflection at the shock, just mechanically. Finally, we perform the return transformation. This sequence changes the velocity! In fact, coming from upstream of the shock the particle gained energy.

Let us have a look at the Earth's bow shock in reality.

Viewgraphs

What are the quantities that we need to measure, when we evaluate such a situation?

Magnetic field B

$$n_1\, v_1\, T_1\, P \longrightarrow \textit{all particle quantities!}$$

We have to measure the distribution function f(v) and then integrate over powers in v to arrive at all "moments" or average values of macroscopic parameters according to:

$\int v^{\boldsymbol{n}}\, f(v)\, \boldsymbol{d}v$ to get n, v, T, p.

We will discuss the necessary instrumentation in the next chapter.